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Fallacies in criticisms of the J-value

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Abstract

Detailed examination of the criticisms of the J-value put forward by Jones-Lee and Chilton shows their points to be without merit. However, the exercise of refuting their critique has brought out a number of J-value implications of potential interest and value to engineering professionals seeking to find the objectively reasonable amount that ought to be spent on a safety system. The paper applies the J-value to the example of a long-term protection system on a notional major-hazard process plant, where a severe accident would otherwise pose a risk of death to the general public either immediately or in the short term. Equations are developed for the improvement in life expectancy produced by averting such an industrial hazard over a prolonged period. The opportunity is taken to review the developments in the J-value that have taken place over the 12 years since the first paper on the method appeared in *Process Safety and Environmental Protection*.

Keywords: J-value; Risk; Immediate hazard; VPF; Safety; Safety fallacies

1. Introduction

The J-value method (J for judgement), first introduced in *Process Safety and Environmental Protection* in 2006 (Thomas et al., 2006a,b,c), estimates the maximum it is reasonable to spend on a safety measure or system by balancing the safety expenditure against the increase life expectancy it brings about. At its core is the concept of the life-quality index (LQI), which is an increasing function of both life expectancy and annual utility, with risk-aversion used to convert monetary income to utility – the satisfaction that the money brings (Nathwani, Lind and Pandey, 1997, Pandey and Nathwani, 2003, Pandey et al., 2006, Nathwani et al., 2009, Thomas et al., 2006a, 2010). The maximum reasonable expenditure will have been reached when the increase in the LQI due to the greater life expectancy the safety measure confers is just matched by the decrease in life quality associated with the fall in income incurred by paying for it.

The J-value, J , is found by dividing the actual cost of the safety measure by the maximum reasonable just found. The starting point for the safety decision is that resources should not be committed in the way proposed if J exceeds 1.0. On the other hand the expenditure will be deemed justifiable if $J \leq 1.0$.

The J-value method is based on actuarial and economic measurements and is fully objective. Importantly, it has been validated against extensive empirical data (Thomas and Waddington, 2017a, Thomas, 2017a).

The objective and validated status of the J-value for assessing the worth of life-extending measures sets it apart from the "value of a prevented fatality" (VPF), a method that has now lost credibility. The figure for the UK VPF was derived from a survey of the opinions of a small sample of people (167) carried out in 1997 (Carthy et al., 1999). The investigators interpreted their survey using a technique of their own devising, namely the two-injury chained method. Wolff and Orr later commented on this procedure (Wolff and Orr, 2009, Appendix 1, Section A.1B):

" the testing of any such assumption, and hence the validation of the method, presents severe challenges. ... In short, if it were possible to validate the chained method, it would not be necessary."

In fact, the surveyors' own data show the two-injury chained method to be invalid (Thomas and Vaughan (2015a)) so that the UK VPF is left without a basis in evidence. The UK Health and Safety Executive (HSE) has been made aware of the problem (Thomas, 2018a), which removes one of the pillars on which safety assessments are made in the UK.

The opinion surveyors have attempted to defend their findings (Chilton et al., 2015), but their points have been refuted (Thomas and Vaughan, 2015b). Two of the surveyors, Jones-Lee and Loomes (2015) essayed a second attempt, but Thomas and Vaughan (2015c) answered their points, concluding:

"The salient fact is that the VPF derived from the two-injury chained method has been shown to be unsubstantiated and it is wholly unsatisfactory that it should be used as a benchmark for safety investment in the UK. We stand by our previous conclusion (Thomas and Vaughan, 2015a) that there is a need for active consideration of methods of valuing human life in the UK that offer an alternative to stated preference techniques. Ensuring that UK workers and public receive adequate protection from industrial and transport hazards means that a re-appraisal is needed urgently of other statistical methodologies that can provide guidance to owners, operators and regulators on decisions on safety."

While Thomas and Vaughan (2015a) provide a comprehensive analysis of the legion of problems besetting the UK VPF, a shorter, easy-to-read commentary on its flaws is given in Thomas and Waddington (2017b).

Owners, operators and regulators seeking guidance for decisions on safety now have the objective and validated J-value method available to them. The J-value has been documented in over 40 published papers (www.jvalue.co.uk) since the original article (Thomas et al., 2006a) appeared in *Process Safety and Environmental Protection* (PSEP). The NREFS study (NREFS, 2017), which explored how best to cope after a big nuclear accident, used the J-value as one of its core techniques. Three independent methods were employed in that multi-university project, and each pointed to the important policy conclusion that mass relocation of people should be

used sparingly if at all, even after a very severe nuclear reactor accident such as Chernobyl or Fukushima Daiichi (Thomas and May, 2017).

It is striking that, despite the very large number of learned-journal papers concerning the J-value published over the years, no comment critical of the J-value appeared in the refereed literature in the first decade after the first article appeared. During that time, the only negative commentary of which we are aware was a review supplied to the Nuclear Division of the Health and Safety Executive by Spackman (2009). Spackman made it clear in his report that he was firmly committed to the UK VPF, a position he reaffirmed afterwards (Spackman et al., 2011). Public comment on his report became possible after it was published openly 2 years later, which led, after correspondence with the Office for Nuclear Regulation (ONR, 2011a,b), to the publication of comprehensive rebuttals from Lind (2011) and from Thomas and Jones (2011). The latter concluded:

"Spackman's review provides a welcome stimulus to discussion on the J-value. However, it suffers from serious omissions and contains inconsistencies, flaws, and disagreements with mainstream economic thinking. In addition, it appears to suggest a bias against innovation, which may act against the overall imperative to ensure that safety related expenditure decisions are rationally and objectively based. It is concluded that the review's multiple deficiencies rule it out as a basis for judgement on the merits or otherwise of the J-value framework."

The tendency to reject novelty identified by Thomas and Jones (2011) would appear to be confirmed by the publication 3 years later of Spackman's "Final comment on the J-value approach" (Spackman, 2014), which includes as its 3rd-last sentence:

"As a valuation method it [the J-value] is much less plausible than those established and developed over recent decades."

viz. the small-scale opinion surveys carried out by Jones-Lee and Chilton in the 1990s (Beattie et al., 1998, Carthy et al., 1999).

But now, for the first time to our knowledge, an article criticising the J-value has appeared in the open, refereed literature (Jones-Lee and Chilton, 2017). It may be noted that Jones-Lee and Chilton are authors of the discredited two-injury chained method used in deriving the UK VPF, and Jones-Lee shares an enthusiasm for the VPF with Spackman, his co-author on several studies (Jones-Lee, Loomes and Spackman, 2007, Spackman et al., 2011, Jones-Lee and Spackman, 2013).

We welcome the contribution of Jones-Lee and Chilton to the public debate on how best to value safety measures. However, the criticisms they put forward appear to be based on fallacious misunderstandings of the J-value, and it is in the public interest that those fallacies should be explored and rebutted.

The paper's format has been chosen to bring out clearly a number of implications of the J-value that are likely to be of interest to users of the method and more generally. To aid readers in their understanding of the issues discussed, a worked example is given for a notional major hazard process plant. The scientific arguments are

presented in Section 2, which takes the form of a distillation of the points contained in Jones-Lee's and Chilton's 2017 paper, followed in each case by a refutation. A discussion will follow in Section 3 and conclusions in Section 4.

As observed above, the comments by Jones-Lee and Chilton concern the earliest J-value paper (Thomas et al., 2006a) and so Appendix A has been included to trace the evolution of the LQI used in the J-value over the intervening 12 years. Then Appendix B explains how the J-value is found from the LQI. Appendix C shows the inappropriateness of the harmonic mean that Jones-Lee and Chilton propose for finding the arithmetic mean of a ratio. Appendix D contains a worked example of a plant falling into the COMAH (Control of Major Accident Hazard) category, and this might usefully be read in advance of Section 2. Appendix E derives, for a prolonged, uniform reduction in the hazard rate, the increase in life expectancy for those in the protected group living at the time the safety measure is introduced. Appendix F derives an equivalent expression for those born into the protected group during the time the safety system is in service. Finally, Appendix G addresses the deficiencies in the argument advanced by Jones-Lee and Chilton against the Total Judgement or J_T -value in the last paragraph of their Section 5.

2. Fallacies and refutations

2.1 Fallacy 1: that the J-value should be regarded as invalid if more life extension is gained by a later, very large reduction in the hazard rate than by a much smaller reduction in risk now

2.1.1 Summary

Jones-Lee and Chilton invite the reader to regard a hazard reduction of 6×10^{-4} over one year for a 40-year-old and a hazard reduction of 240×10^{-4} over 12 months for an 80-year-old as equivalent, despite the second reduction being 40 times greater. The suggestion may represent a departure from common sense – why should anyone think that very dissimilar risk reductions ought to be regarded as comparable? – but it constitutes the "principal" component of Jones-Lee's and Chilton's case against the J-value.

Having arranged for the risk reduction offered at age 80 to be a great deal larger than that offered at age 40, Jones-Lee and Chilton find, not surprisingly, that the former gives a superior increase in life expectancy. They then suggest that the J-value would favour the offer at age 80 over that available at age 40. This is entirely incorrect.

In fact, Jones-Lee and Chilton commit the elementary economic blunder of ignoring the cost side of the cost-benefit balance. Removing a 40 times bigger risk will actually cost a lot more. This will render the J-value for the immediate offer to the 40-year-old significantly lower than the J-value for the risk reduction offered at a later age. Hence not only would the J-value favour immediate risk reduction if the reduction were to be the same at age 80 as at age 40, but the J-value would also favour immediate risk reduction in the case proposed by Jones-Lee and Chilton, where those authors chose the later risk reduction to be hugely greater.

What the J-value would recommend can thus be seen to be the exact opposite of what Jones-Lee and Chilton allege.

2.1.2 Detailed analysis

The main aim of Jones-Lee's and Chilton's paper is set out in the concluding two sentences of their Section 1:

"Although Spackman does not spell this out, one such factor is clearly the nature of the small change in the survival function that gives rise to the change in life expectancy. The principal purpose of this paper is to deal in more detail with this particular limitation of the J-value model ..."

This "principal purpose" is then picked up in Section 3, where the claim is made, in paragraph 2, that the J-value is

"incapable of taking account of the fact that a given gain in life expectancy can, in principle, be generated by any one of a variety of different types of perturbation, or very small changes, in an individual's survival function (or, equivalently, her vector of future hazard rates)"

This is a puzzling statement. It is well known that the life expectancy, $X(a)$, at age, a , is the integral over all future ages, of the survival probabilities, $S(t|a)$, to age, t , given that age, a , has been reached and that, moreover, these survival probabilities will depend on the integral of the hazard rate, $h(u)$, for future years, u . Thus

$$X(a) = \int_{t=a}^{\infty} S(t|a) dt = \int_{t=a}^{\infty} e^{-\int_{u=a}^t h(u) du} dt \quad (1)$$

See, for example, Marshall et al. (1983). Meanwhile a life extending measure will lead to a reduction in hazard rate, $b(u)$, which can be in force for many years, during which the hazard reduction may vary with time. Life expectancy will then increase to:

$$X'(a) = \int_{t=a}^{\infty} S'(t|a) dt = \int_{t=a}^{\infty} e^{-\int_{u=a}^t (h(u)-b(u)) du} dt \quad (2)$$

The gain in life expectancy may then be found by subtraction.

It is not clear what point Jones-Lee and Chilton were hoping to make in the second claim cited above, but equation (2), used in J-value analysis, demonstrates that the statement is clearly untrue. J-value calculations can be carried out for any small variation to the hazard rate, including a time-varying perturbation that lasts for a prolonged period. This feature of J-value analysis, which is explained in more detail in Section 2.4, allows an objective assessment to be made of the economic worth of a safety measure or a protection system guarding against any time-profile of hazard to

human life. A recent example is the evaluation of the recommendations to relocate for people living near the power plants after the big nuclear accidents at Chernobyl and Fukushima Daiichi (Waddington et al., 2017). Here the J-value analysis was able to account for a time-varying radiation hazard that took 70 years to decay to a low level.

Jones-Lee and Chilton attempt to clarify their position by devoting their next paragraph to a hypothetical game (which they make no attempt to relate to reality):

"Thus, for example, consider a situation in which an individual aged 40 is informed that in the UK for a person of his/her age the probability of death during the coming year is about 14 in 10,000, while if he/she were to survive to the age of 80 the probability of death during the coming year would rise to roughly 660 in 10,000. Suppose that the individual is then asked which of the two risks he/she would prefer to have halved. In spite of the fact that the probability of the 40-year old surviving to age 80 is only about 0.63, his/her gain in undiscounted life expectancy resulting from a halving of the hazard rate at age 80 would actually be about *six times larger* than the gain resulting from a halving of the risk of death during the coming year"
[The italics are those of Jones-Lee and Chilton.]

No algebra is provided by Jones-Lee and Chilton to justify the point being made, but it is desirable, in the interests of fairness, that the reader should understand their various claims in sufficient detail to be able to judge them. Therefore we offer the following explanation of the route that presumably those authors took to arrive at their figures.

Suppose that the hazard rate at age, a , is subject to a uniform reduction over a period of one year, so that the perturbation to the hazard rate takes the form:

$$b(u) = \begin{cases} b & \text{for } a < u \leq a+1 \\ 0 & \text{for } u > a+1 \end{cases} \quad (3)$$

where $b(u)$ depends on age, u , while b on its own is independent of u .

[The stipulation by Jones-Lee and Chilton of a duration of one year seems to arise because of those authors' promotion of the mistaken view that it is invalid to calculate the gain in life expectancy that a protection system will bring except over the coming year:

"it is necessary to ensure that the gains in life expectancy being valued are the result of reductions in the risk of death during the coming year, rather than delayed risk reductions that will be effective only in later years of life"

Paragraph 4 of Section 1 of Jones-Lee and Chilton (2017).

While, as will now be shown, the gain in life expectancy can be estimated in a particularly simple way when the hazard is reduced for just one year, it is eminently possible to find the change in life expectancy when the reduction in hazard rate persists over many years. See Section 2.4.]

Substituting from equation (3) into equation (2) gives the increased life expectancy at age, a , as:

$$\begin{aligned} X'(a) &= \int_{t=a}^{\infty} e^{-\int_{u=a}^t (h(u)-b(u))du} dt = \int_a^{a+1} e^{-\int_a^t h(u)du} dt = \int_a^{\infty} e^{-\int_a^t h(u)du} dt \\ &= e^b \int_a^{\infty} e^{-\int_a^t h(u)du} dt = e^b X(a) \approx (1+b) X(a) \end{aligned} \quad (4)$$

where the last step has used the expansion, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, truncated after the first two terms, a reasonable approximation when x is small. Subtracting the original life expectancy at age a , $X(a)$, then gives the change as:

$$\begin{aligned} \delta X(a) &= X'(a) - X(a) \\ &\approx bX(a) \end{aligned} \quad (5)$$

It will be shown in Section 2.2 that Jones-Lee and Chilton are incorrect to assume that the J-value should apply to individuals within the group to be protected rather than to the group as a whole. However we can get around this point for the present purposes by assuming that the population to be protected consists of a group of individuals of the same age, or else by making the assumption the group consists of a single member of age 40. Adopting the second course, let us imagine that the sole member of the group, an individual of age 40 is offered the choice between a reduction in hazard rate over the coming year and a similar reduction over 12 months in 40 years' time, if and when he/she has attained the age of 80. Continuing this line of thought, it is possible to use equation (5) to calculate the change in life expectancy for the individual of age 40 as

$$\delta X(40) \approx bX(40) \quad (6)$$

Considering life expectancy at age 80, if the individual dies before the age of 80, he/she will experience no change in life expectancy at that age. On the other hand, if he/she lives to 80, he/she will gain $\sim bX(80)$ in life expectancy. The chance of the latter gain in life expectancy being achieved is the probability of living to age 80 given survival to age 40, namely $S(80|40) = S(80)/S(40)$, the right-hand side of which equality comprises the ratio of the survival probabilities to ages 80 and 40 respectively. Hence the expected value, $\delta X(80|40)$, of the gain in life at age 80 resulting from a year's reduction in hazard 40 years from now for the individual currently aged, 40, may be written:

$$\begin{aligned}\delta X(80|40) &\approx S(80|40) \times bX(80) + (1 - S(80|40)) \times 0 \\ &\approx \frac{S(80)}{S(40)} \times bX(80)\end{aligned}\quad (7)$$

It is, of course, possible to generalise equation (7) for any $t : t \geq 40$ to give the improvement in life expectancy for a reduction, b , in the hazard rate occurring at later age, t :

$$\delta X(t|40) \approx S(t|40) \times bX(t) \quad (8)$$

If we follow Jones-Lee and Chilton and assume that b is 50% of the background hazard rate at age 40, then we may calculate b as: $\frac{1}{2} \times 1.247 \times 10^{-4} = 6.24 \times 10^{-4}$. On inserting this figure into equation (8), and using the values for $S(40)$, $S(80)$ and $X(80)$ given in Table 1, the resulting changes in life expectancy may be calculated as

$$\begin{aligned}\delta X(40) &= 0.026 \text{ y} = 9.57 \text{ days} \\ \delta X(80|40) &= 0.0035 \text{ y} = 1.29 \text{ days}\end{aligned}\quad (9)$$

[Jones-Lee and Chilton do not specify a life table to which they applied their method, saying only that "These are the approximate UK hazard rates averaged over males and females" (their end-note 7). In the interests of accuracy, consistency and traceability, actuarial figures used in this paper will be based on combined gender data from the most recent life tables (2014 – 2016) available for the UK at the time of writing (Office of National Statistics, 2017)].

Applying equation (8) to all ages up to 100 results in Figure 1, which shows that the gain in life expectancy falls roughly linearly with age, and that the gains decline to nearly zero for future ages in excess of about 90 years.

Inspection of the falling curve of Figure 1 prompts the question: how was it that Jones-Lee and Chilton managed to suggest that the improvement in life expectancy for implementation of the health and safety measure 40 years later at 80 rather than now, at age, 40, could be "*six times larger*" (their italics) ?

The trick is to multiply the hazard reduction offered at age 40 by a very large factor prior to making the proposal for age 80. No explicit justification is given by Jones-Lee and Chilton for reducing the hazard tens of times more at age 80 than at age 40. While the statement is made that the background hazard rate is reduced by 50% at each age, no attempt is made to explain why this much greater reduction in hazard rate at age 80 should be regarded as comparable with the smaller decrease at age 40.

It will, of course, be evident even to the casual reader that the background hazard rate for a person of 80 must be very much higher than that facing a person of 40 simply because of the natural effects of old age. But why should this disparity be reflected in the offers made? In fact, Jones-Lee and Chilton suggest the ratio of hazard reductions

should be $660 \times 10^{-4} / 14 \times 10^{-4} \approx 47$, so that the reduction in hazard offered at age 80 is nearly 50 times higher than the reduction at age 40. The actuarial figures for 2014 – 2016 suggest that the background hazard rates are 12.47 in 10,000 at age 40 and 488 in 10,000 at age 80, which imply a slightly lower but still very high ratio, viz. $488 \times 10^{-4} / 12.47 \times 10^{-4} \approx 39$.

Reductions in hazard rate of $b = 6.24 \times 10^{-4}$ at age 40, and a 39 times greater figure at age 80, namely $b = 244 \times 10^{-4}$, produce the following expected values:

$$\begin{aligned} \delta X(40) &= 0.026 \text{ y} = 9.57 \text{ days} && \left(\text{as before with } b = 6.24 \times 10^{-4} \right) \\ \delta X(80|40) &= 0.137 \text{ y} = 50.2 \text{ days} && \left(\begin{array}{l} \text{where the previous value of } b \text{ has been} \\ \text{multiplied by 39: } b = 244 \times 10^{-4} \end{array} \right) \end{aligned} \quad (10)$$

The first figure, 9.57 days, is similar to that quoted by Jones-Lee and Chilton (0.028 y = 10.2 days), while the second is a little less than their figure of 0.166 y = 60.6 days. Nevertheless equation set (10) reproduces a substantially higher improvement in life expectancy at age 80 than at age 40, as reported by Jones-Lee and Chilton (2017). At this point those authors assert that

"according to the J-value model, the individual would express a clear preference for the delayed risk reduction."

which claim they assert as evidence for a "fundamental conceptual limitation" in the J-value.

To be clear, what Jones-Lee and Chilton are alleging is that the 40 year old, guided by the J-value, would reject a reduction in hazard now in favour of a 39 times greater reduction in risk in 40 years' time if and when he/she has reached the age of 80. Patently, Jones-Lee and Chilton have provided a huge (and unexplained) weighting to make the later hazard reduction look more attractive. But it turns out that, even so, the allegation that the J-value would favour the delayed reduction in risk is false.

Jones-Lee and Chilton do not present a J-value calculation to back up their assertion. But clearly it is only by so doing that an assessment can be made of the relative worth of the increases in life expectancy given in equation set (10), as measured by the J-value. So let us now use the J-value to examine Jones-Lee's and Chilton's rather bold claim.

The general equation for the J-value is derived in Appendix B as equation (B.4), repeated below for the convenience of the reader

$$J = \frac{1-\varepsilon}{G} \delta \hat{G} \frac{X}{\delta X} \quad (\text{B.4})$$

where ε is the risk-aversion, G is the gross domestic product (GDP) per head, $\delta \hat{G}$ is the annual cost of the protection measure per protected person, X is the life

expectancy of the population group under consideration and δX is the improvement in life expectancy brought about by the safety measure. Suppose that the annual cost of the health and safety measure needed to reduce the hazard rate by $b = 6.24 \times 10^{-4}$ is $\delta \hat{G}_1$. The J-value, J_1 , associated with applying this hazard rate reduction at age 40, is:

$$J_1 = \frac{1-\varepsilon}{G} X(40) \delta \hat{G}_1 \frac{1}{\delta X(40)} \quad (11)$$

Similarly, the J-value, J_2 , associated with applying the hazard rate reduction, at age 80 will be

$$J_2 = \frac{1-\varepsilon}{G} X(40) \delta \hat{G}_2 \frac{1}{\delta X(80|40)} \quad (12)$$

where $\delta \hat{G}_2$ is the annual cost per protected person of the health and safety measure needed to reduce the hazard rate by $b = 244 \times 10^{-4}$ and $X(40)$ is retained in equation (12) because it is the 40 year old who is taking the decision.

Knowledge of the relationship between $\delta \hat{G}_1$ and $\delta \hat{G}_2$, would give us an idea of how to assess the two proposals by applying the J-value.

It is clear that reducing the background hazard rate by as much as 50% for a 40 year old is likely to be difficult. It might involve, for example, isolating that person from accident hazards and sources of infection, while providing the highest level of medical care on immediate call and perhaps instituting a suicide watch. But reducing the background hazard rate for the 80 year old by the far greater amount needed to halve his/her hazard rate would obviously be a significantly more difficult undertaking, simply because so much of the hazard faced by an 80 year old is associated with old age and so intrinsic and very difficult to influence.

In reality, achieving the greater reduction in hazard rate specified for the 80 year old is likely to be impossible, and our best model is then to assign an infinite cost to the process: $\delta \hat{G}_2 \rightarrow \infty$. This would imply, from equation (12), that the J-value for this option, J_2 , would tend to infinity. This would mean that the alternative choice of an immediate reduction in hazard rate at age 40 would be preferable for any finite value of implementation cost, $\delta \hat{G}_1$.

At the very least, the law of diminishing returns would surely be a large factor in the cost of the very great reduction in hazard rate specified for the 80 year old. But, in a spirit of indulgence, let us assume that $\delta \hat{G}_2$ remains finite and that, moreover, cost is proportional to the reduction in hazard rate. Under this assumption,

$$\frac{\delta\hat{G}_2}{\delta\hat{G}_1} = \frac{b_2}{b_1} \quad (13)$$

where $b_1 = 6.24 \times 10^{-4}$ is the hazard reduction offered at age 40 and $b_2 = 244 \times 10^{-4}$ is the hazard reduction offered at age 80.

Let us now assume further that the proposition to the 40 year old is that the reduction in hazard rate 40 years later at the age of 80 is to be accompanied by an offer to discount back to today the cost of the later reduction using the social discount rate. Using the results of Thomas and Waddington (2017a), this can be estimated as the growth in GDP per head, found to be 2.03% per annum for the UK for the period 1961 to 2013 (World Bank, 2017). Hence $\delta\hat{G}_2$ in equation (12) is replaced by a lower, discounted version, $\delta\hat{G}_{2d}$. This will tend to increase the attraction of the delayed version.

When this is all done, the ratio of the J-values may be found by dividing equation (12) by equation (11):

$$\begin{aligned} \frac{J_2}{J_1} &= \frac{\delta\hat{G}_{2d}}{\delta\hat{G}_1} \frac{\delta X(40)}{\delta X(80|40)} = \left(\frac{1}{1+r^*} \right)^{40} \frac{\delta\hat{G}_2}{\delta\hat{G}_1} \frac{\delta X(40)}{\delta X(80|40)} \\ &= \left(\frac{1}{1+r^*} \right)^{40} \frac{b_2}{b_1} \frac{\delta X(40)}{\delta X(80|40)} \approx \left(\frac{1}{1+r^*} \right)^{40} \frac{b_2}{b_1} \frac{S(40)}{S(80)} \frac{b_1 X(40)}{b_2 X(80)} \\ &\approx \left(\frac{1}{1+r^*} \right)^{40} \frac{S(40)}{S(80)} \frac{X(40)}{X(80)} \approx \left(\frac{1}{1.0203} \right)^{40} \times \frac{0.9809}{0.6203} \times \frac{42.03}{8.92} = 3.33 \end{aligned} \quad (14)$$

where discounting at the social discount rate, r^* , is applied to $\delta\hat{G}_2$ in the second step, equation (13) is used in the third and equation (8) in the fourth.

Based on this J-value analysis and making generous concessions to boost the attractiveness of delaying the risk reduction, the option of taking the immediate reduction in hazard rate at age 40 still emerges as over three times as cost-beneficial. For example, if the J-value at age 40 were unity, the individual might well decide to accept that option while, by the same token, rejecting the risk-reduction scheme to be applied at age 80 as over 3 times too expensive because $J = 3.33$.

Clearly Jones-Lee and Chilton are entirely wrong to suggest that the J-value would favour choosing the delayed option. They have committed the elementary economic error of paying no attention to the cost side of the cost-benefit equation.

Moreover, while those authors took pains to offer in their hypothetical game a much greater reduction in hazard rate at age 80 than at age 40, in fact this boosting of the benefits of the delayed option is a matter of no consequence. Inspection of equation (14) shows that, when the cost of the benefit is taken, generously to the Jones-Lee and Chilton argument, to be proportional to the size of the hazard reduction, the degree by which the hazard is reduced has no influence on the ratio of the J-values. While

Jones-Lee and Chilton set the reduction in hazard rate at age 80 nearly 50 times the reduction they offered immediately to the middle aged person, they could as well have set the octogenarian's reduction even higher and it would have made no difference.

Of course, in reality, the law of diminishing returns would mean that the 50 times greater reduction in hazard rate offered at the later age would attract much more than 50 times the cost. This large cost premium would only strengthen the relative attractiveness of immediate over delayed implementation for the protection measure.

Thus the true conclusion to be drawn from Jones-Lee's and Chilton's hypothetical game is the exact opposite of the one that they present as their main argument against the J-value. The J-value will favour risk reduction now over risk reduction in 40 years' time. Thus Jones-Lee's and Chilton's "principal" objection may be dismissed as poorly thought out and wrong.

2.2 Fallacy 2: that the J-value should be applied on an individualistic basis

Although Jones-Lee and Chilton do not make it clear that they are making a new (and incorrect) assumption, the mistaken idea that the J-value should apply on an individualistic basis rather than at the group level seems to underlie several of the misunderstandings exhibited in their paper. The plainest statement of their implicit viewpoint occurs in paragraph 3 of their Section 4, where they argue against the averaging process inherent in the LQI and the J-value. Instead they appear to favour dealing with individualised variables. Their implied recommendation is then that the J-value should be the average of all the individual J-values in the population to be protected. It will be shown that such a recommendation is ethically incorrect and at variance with the way that decisions on life extending activities are taken across the world.

A derivation of the J-value is given in Appendix B, where it is made plain that population-averaged parameters are used (as, of course, did Thomas et al., 2006a, as well as succeeding papers). That this is the morally correct thing to do can be seen through examining the case where J takes its limiting desirable value, $J = 1$, so that the actual expenditure and the ideal maximum spend are equal. Combining equations (B.2) and (B.3) at $J = 1$ gives

$$\delta\hat{G}\Big|_{J=1} = -\frac{G}{1-\varepsilon} \frac{\delta X}{X} \quad (15)$$

where $\delta\hat{G}\Big|_{J=1}$ is the annual cost per person of the safety measure at $J = 1$, G is the average income in the population to be protected, taken for ethical reasons to be the Gross Domestic Product (GDP) per head, ε is the risk-aversion, found equal to 0.91 for developed countries (Thomas and Waddington, 2017a), X is the population-average life expectancy and δX is the population-average improvement in life expectancy brought about by the safety measure. The minus sign preceding the expression on the right-hand side of equation (15) indicates that the annual cost of the safety measure reduces the income per head.

The change in population-average life expectancy, δX , is found as

$$\delta X = \sum_{k=a_0}^{a_f} \frac{n_k}{n_T} \delta X_k \quad (16)$$

where δX_k is the average increase in life expectancy conferred by the safety improvement on someone of age, k ; n_k is the number of people in the population with age, k ; a_0 is the lowest age in the population and a_f is the highest. Meanwhile n_T is the total number in the population, across all ages:

$$n_T = \sum_{k=a_0}^{a_f} n_k \quad (17)$$

Meanwhile the relationship between the average change in life expectancy at age, $k : \alpha_0 \leq k \leq \alpha_f$, and the change, δX_{kj} , experienced by each individual, $j : j = 1, 2, \dots, n_k$, of that age is:

$$n_k \delta X_k = \delta X_{k1} + \delta X_{k2} + \dots + \delta X_{kj} + \dots + \delta X_{kn_k} \quad (18)$$

Substituting Equation (18) into Equation (16) gives:

$$\delta X = \frac{1}{n_T} \left(\begin{array}{l} \delta X_{a_0 1} + \delta X_{a_0 2} + \dots + \delta X_{a_0 n_{a_0}} + \delta X_{a_0+1,1} + \delta X_{a_0+1,2} + \dots + \delta X_{a_0+1, n_{a_0+1}} + \dots \\ \delta X_{k,1} + \delta X_{k,2} + \dots + \delta X_{k, n_k} + \dots + \delta X_{a_f 1} + \delta X_{a_f 2} + \dots + \delta X_{a_f n_{a_f}} \end{array} \right) \quad (19)$$

Finally, substituting from equation (19) into equation (15) gives:

$$\delta \hat{G} \Big|_{J=1} = - \frac{1}{n_T} \frac{G}{1-\varepsilon} \frac{1}{X} \left(\begin{array}{l} \delta X_{a_0 1} + \delta X_{a_0 2} + \dots + \delta X_{a_0 n_{a_0}} \\ + \delta X_{a_0+1,1} + \delta X_{a_0+1,2} + \dots + \delta X_{a_0+1, n_{a_0+1}} + \dots \\ + \delta X_{k,1} + \delta X_{k,2} + \dots + \delta X_{k, n_k} + \dots \\ + \delta X_{a_f 1} + \delta X_{a_f 2} + \dots + \delta X_{a_f n_{a_f}} \end{array} \right) \quad (20)$$

Equation (20) is important as it shows that calculating the ideal maximum annual payment using the J-value causes the gain in life expectancy of each person in the group to be given the same weighting. Formally,

$$\frac{\partial}{\partial \delta X_{kj}} \left(\delta \hat{G} \Big|_{J=1} \right) = - \frac{1}{n_T} \frac{G}{1-\varepsilon} \frac{1}{X} \quad \text{for all ages, } k, \text{ and individuals, } j \quad (21)$$

Thus equation (20) embodies the ethical position that, under the J-value method, each day of life extension is valued the same for each member of the population, rich or poor, young or old. It is exactly this moral and philosophical position that is argued for from a legal perspective by Sunstein (2003), who concludes

"a focus on statistical life-years has an important kind of neutrality: It treats everyone's life-years the same. ... it is better to attend to statistical life-years than to statistical lives. If either approach discriminates, it is one that relies only on statistical lives, because that approach treats the life-years of older people as worth more than the life-years of younger people."

Moreover, while the LQI valuation began as 'normative', in the economic sense of carrying a moral endorsement (*viz.* equal treatment for citizens of all ages and incomes), the J-value embodying the LQI principle is capable of producing an economically 'positive' statement that can be put to the test. The success achieved in that test (Thomas and Waddington, 2017a) implies that, when it comes to decisions on life extending decisions, people's behaviour in nations all over the world is described well by the J-value.

Hence Jones-Lee's and Chilton's argument against the averaging used in the J-value falls both ethically (Sunstein, 2003) and empirically (Thomas and Waddington, 2017a). The fact that the validated J-value method applies to groups of people rather than to individuals invalidates a number of other peripheral objections raised by Jones-Lee and Chilton such as their objection to the use of "UK national averages for life-expectancy" (paragraph 3 of their Section 4) and their advocacy of a separate rate of time preference for each individual as opposed to a social discount rate (paragraph 1 of their Section 5 and their end-note 13).

2.2.1 Calculating $J(a)$

Even though the use of individualistic J-values transgresses the requirements of ethics, it is nevertheless interesting to explore, as an exercise, what would happen if the J-value were to be taken as an average of constituent J-values. It will be shown, using the example of Appendix D, that the effect is likely to be relatively small.

Note first that the J-value is given by equation (B.4) of Appendix B, repeated below:

$$J = \frac{1-\varepsilon}{G} \delta \hat{G} \frac{X}{\delta X} \quad (\text{B.4})$$

Two of the variables in equation (B.4), X and δX , are dependent on age, a , so that we could define an age-specific J-value as:

$$J(a) = \frac{1-\varepsilon}{G} \delta \hat{G} \frac{X(a)}{\delta X(a)} \quad (22)$$

which might then be regarded as applying to individuals of age, a . Applying the expectation operator, $E(\cdot)$, produces:

$$E(J(a)) = \frac{1-\varepsilon}{G} \delta \hat{G} \times E\left(\frac{X(a)}{\delta X(a)}\right) \quad (23)$$

Jones-Lee's and Chilton suggestion that the harmonic mean of the denominator of the expectation term should be used at this point is incorrect, as explained in Appendix C. An approximate analytic method based on a Taylor series expansion is sometimes used to find the expectation of a ratio of random variables (e.g. Rice, 2007, p. 160), but a more accurate result can be achieved numerically when the details of a specific application of the J-value is available, such as provided by Appendix D.

Appendix D details the case of a process plant where a safety system with a service lifetime of 25 years is under consideration. The J-value under strong intergenerational equity is given by $J = 0.58$, with $J < 1.0$ implying endorsement for the installation of the protection system under consideration. Little difference to the J-value occurs under the conditions of weak and mixed intergenerational equity. See Table 2. (See Appendix B, Sections B.2 and B.3 for a discussion of the concept of intergenerational equity and its strong, weak and mixed forms in the context of the J-value.)

Age-specific J-values, $J(a)$, may be calculated for this example, and these are plotted against age, a , in Figure 1, which also displays the actual J-value marked on as the horizontal line. (Negative ages refer to people yet to be born at the time of installation of the safety system.) It is clear from Figure 1 that $J(a)$ stays close to the J-value for the greatest part of the age range, for $-10 \leq a \leq 60$. But $J(a)$ rises at both ends of the age interval, with the growth becoming increasingly marked towards the extremities of the range.

Older people getting closer to the end of their days at the time of installation are increasingly likely to have too little time left to gain full advantage from the system, and so their personal J-value will be high. What would have been strongly in their personal interest 20 or 30 years ago, when $J(a) \approx J$, will become less so as they age, with, in the case under consideration, $J(a) > 1$ for $a > 70$.

The situation with those yet to be born is somewhat different. Because of the low level of the natural hazard rate facing the young, they will almost all ($S(25) > 0.99$) experience the benefit the protection offers for the full period from their date of birth to the date when the protection system is retired at the end of its 25 year lifetime. The only reason such young people should regard the system as poor value for money is that the protection system comes out of service too soon, before they reach the age of 25, before they will have experienced its full benefit. If a similar new system with more service time ahead of it were on offer as a follow-on replacement, it would be in their interest (in the sense that $J(a) < 1$) to have it installed in place of the old safety system when the first system comes to the end of its life.

The average value, $E(J(a)) = 0.93$, for the worked example turns out to be 60% higher than the actual J-value based on strong intergenerational equity: $J = 0.58$. Thus it constitutes an approximation to the J-value, but one that would be somewhat biased against the implementation of the protection measure.

In fact, the measure, $E(J(a))$, fails the ethical criterion suggested by Sunstein (2003) that each person's next day of life should be valued the same, rich or poor, old or young. There is, of course, nothing to stop anyone calculating $J(a)$ on a personal basis, and such a calculation might, indeed, provide that person with some guidance. However, it is not a suitable basis for spending decisions taken on behalf of the group to be protected.

2.3 Fallacy 3: that the J-value is a step on the way to finding an improved estimate of the "value of a prevented fatality" (VPF)

Jones-Lee and Chilton devote a significant part of their critique of the J-value trying to defend the UK VPF and their derivation of it. Despite their claim that their work has been subject to "much scrutiny by the Health and Safety Executive" and to "ongoing review" in a document co-authored by Jones-Lee and Spackman, they have failed to answer the important criticisms of the VPF that have appeared in the scientific literature. Jones-Lee and Chilton express the hope in their end-note 14 that:

"in Chilton et al. (2015) it is shown that Thomas and Vaughan's (2014 [Thomas and Vaughan, 2015a]) criticisms are, in the main, ill-founded and somewhat spurious."

Sadly for them, this statement is entirely inaccurate. The multiple flaws in their interpretation of their data revealed in Thomas and Vaughan (2015a) are fundamental and invalidate their results. The two attempts to defend their methodology (Chilton et al., 2015, Jones-Lee and Loomes, 2015) were refuted comprehensively by Thomas and Vaughan (2015b,c). It is thus clear that the VPF in use for the best part of the last 20 years in the UK has not been fit for purpose.

Jones-Lee and Chilton are clearly happiest talking about the VPF and they spend time attempting to derive a VPF that they claim is implied by the J-value. They also devote effort to devising constraints on how (and, apparently, how not) a gain in life expectancy can be translated into a VPF (paragraph 4 of their Section 1). Such extensive concern with the VPF in a paper apparently discussing the J-value is odd, and gives the erroneous impression that the J-value is a step on the way to deriving a VPF.

The stance is particularly strange when the proponents of the J-value have made it fully clear that the VPF can only ever be a crude measure for the valuation of human life. Detailed criticisms of the concept of a VPF have appeared in, for example, Thomas and Vaughan (2013), Thomas and Waddington (2017b), Thomas (2017b) and Thomas (2017c). These criticisms are additional to the demonstration (Thomas and Vaughan, 2015a, b, c) of the invalidating flaws associated with the derivation of the VPF used in the UK (Carthy et al., 1999).

Given the apparent confusion on the part of Jones-Lee and Chilton, we state here, for the avoidance of doubt that the J-value does not use the VPF and that it is not part of the J-value's purpose to find a VPF, which is a poor measure of human harm.

The flexibility of the J-value approach makes it possible, of course, to extrapolate from J-value results to estimate a value of life-to-come for the average person in the population. But it is certainly not the case, as hopefully suggested by Jones-Lee and Chilton that

"the J-value model actually produces an implied VPF that is very much closer to the DfT figure"

Jones-Lee and Chilton, Section 5, paragraph 1

Carrying out the exercise shows how the UK VPF is set anomalously low (Thomas, 2018). This is a worrying situation for UK citizens when it is realised that the Government makes extensive use of the UK VPF as a spending criterion in a large number of its departments and agencies (Deloitte, 2009).

2.4 Fallacy 4: that it is invalid to put a value on the gain in life expectancy unless it results from a reduction in hazard rate confined to the next year

By cloaking their comments in the straw-man proposition that the J-value is intended to be a step on the way to finding an improved figure for the VPF, Jones-Lee and Chilton may have created the erroneous impression that it is in some way invalid to calculate the gain in life expectancy that a protection system will bring except over the coming year:

"it is necessary to ensure that the gains in life expectancy being valued are the result of reductions in the risk of death during the coming year, rather than delayed risk reductions that will be effective only in later years of life"

Section 1, paragraph 4, Jones-Lee and Chilton (2017).

A similar idea reappears in the first sentence of the last paragraph of Section 3:

"In short, the fundamental conceptual limitation of the J-value model is that it focuses exclusively on the increase in life expectancy resulting from a reduction in the hazard rate for any given year"

This statement is puzzling. Even a cursory look just the titles of published papers given on www.jvalue.co.uk would have revealed this to be a ludicrous claim, as the J-value has always been applied to assess protection systems with mission times that could be years or decades long. Papers listed on that easy-to-google website include:

- "The life extension achieved by eliminating a prolonged radiation exposure" (Thomas et al., 2006c)
- "Analytical techniques for faster calculation of the life extension achieved by eliminating a prolonged radiation exposure" (Thomas et al., 2007a),
- "Numerical techniques for speeding up the calculation of the life extension brought about by removing a prolonged radiation exposure " (Jones et al., 2007),
- "Calculating the benefit to workers of averting a radiation exposure lasting longer than the working lifetime" (Thomas and Jones, 2009).

Nevertheless, Jones-Lee and Chilton claim there are "clear theoretical and empirical grounds" for their view that the change in hazard rate should be confined to one year,

and quote, among others, Johannesson et al. (1997). The reference to Johannesson et al. (1997) is striking because in their study, those authors calculate the total change in life expectancy caused by a small reduction in the hazard rate that lasts for much longer than "the coming year" stipulated by Jones-Lee and Chilton. In fact, the reduction persists for the rest of the person's life ("permanent change in the hazard rate"). Johannesson et al. then produce a "value of a statistical life" from their estimate of the worth of the total change in life expectancy. Dividing this monetary sum by the change in life expectancy to give the value of a life year, they then multiply by the expected life to come. The value of life is thus made proportional to life expectancy, a reasonable proposition that is, of course reflected in the LQI and the J-value (see equations (A.1) and (A.5)).

Thus, while the work by Johannesson et al. is quoted in aid by Jones-Lee and Chilton, far from confirming their argument, the Johannesson study runs directly counter to it.

In fact, Johannesson et al. (1997) were not entirely happy with their results, which rested on an opinion survey. Interestingly, they commented in the last paragraph of their Concluding Remarks: "Another possibility is that our valuation question did not work" before ending with the final words: "our results should nevertheless be interpreted with caution" – a frank acknowledgement of the difficulties associated with stated preference methods. For a further discussion of the problems in obtaining a meaningful answer from an opinion survey, see Thomas (2018c).

To clear up any lingering anxiety that it might be invalid to use the gain in life expectancy unless it results from a reduction in hazard rate confined to the next year, Appendices E and F derive expressions for change in life expectancy for the industrially relevant case where the safety system or measure brings a uniform and immediate reduction in hazard rate throughout the time, T_s , it is in service, which may be several decades. Appendix E details the derivation for the members of the public in the protected group who are living at the time of installation or initiation of the safety measure. Appendix F does the same for the members of the protected group who are born during the service time of the protection measure. The overall change of life expectancy, δX_p , for the combined cohort being protected is then given by equations (F.11), (E.16) and (F.10), repeated below:

$$\delta X_p = \frac{X(0)\delta X_1 + T_s\delta X_2}{X(0) + T_s} \quad (F.11)$$

where

$$\delta X_1 = \frac{1}{X(0)} \left(\int_{a=0}^{\alpha_0 - T_s} \int_{t=a}^{a+T_s} (e^{b(t-a)} - 1) S(t) dt da + \int_{a=0}^{\alpha_0 - T_s} \int_{t=a+T_s}^{\alpha_0} (e^{bT_s} - 1) S(t) dt da \right. \\ \left. + \int_{a=\alpha_0 - T_s}^{\alpha_0} \int_{t=a}^{\alpha_0} (e^{b(t-a)} - 1) S(t) dt da \right) \quad (E.16)$$

and

$$\delta X_2 = \frac{1}{T_S} \left(\int_{\tau=0}^{T_S} \int_{t=0}^{T_S-\tau} S(t) (e^{bt} - 1) dt d\tau + \int_{\tau=0}^{T_S} \int_{t=T_S-\tau}^{\alpha_0} (e^{b(T_S-\tau)} - 1) S(t) dt d\tau \right) \quad (F.10)$$

The above equations are clearly significantly more complex than equation (5), which represented the special case where the protection system had a service lifetime of only one year. Moreover the absence of an analytic expression for $S(t)$ means that the integrations need to be performed numerically. But the equations give a clear demonstration of how it is possible to calculate the change in life expectancy for the protected population as a whole over a prolonged period, during which the protection system is in service.

Equations (E.16) and (F.10) for δX_1 and δX_2 apply for the case where the hazard is fatality in the immediate or short-term future. The important case of nuclear radiation, where any possible death from a radiation-induced cancer may be delayed for decades after induction, is covered in the references previously cited in the bulleted list above.

2.5 Fallacy 5: that the J-value implies that the individual judges whether or not to pay his/her share of a protection system based on his/her certain knowledge of the dates he/she will die with and without the safety system

This perplexing suggestion is contained in the closing sentence the last paragraph of Section 3 of Jones-Lee and Chilton (2017):

"In short, there is nothing that distinguishes the situation modelled by Thomas et al. (2006) from the completely unrealistic scenario in which an individual knows *for certain* how long he/she will continue to survive and must then decide how much he/she is willing to pay per annum for a marginal 'end-of-life' addition to that known remaining survival time."

[The italics are those of Jones-Lee and Chilton.]

The claim reappears in the first sentence of Section 4, presumably under the assumption that the prior assertion of this non-sequitur in Section 3 was enough to justify it:

"Given its failure to take adequate account of the effect of uncertainty concerning the time of death, the J-value model can therefore hardly be regarded as providing a satisfactory basis for the quantitative analysis of individual attitudes to physical risk."

In fact the J-value paper (Thomas et al., 2006a) cited by Jones-Lee and Chilton pays clear and obvious attention to the stochastic nature of the threats under consideration. The theme is developed further in the companion papers carried by *Process Safety and Environmental Protection* on consecutive pages (Thomas et al., 2006b,c). Therefore it seems odd, to say the least, that such an opinion could be formed and seriously advanced.

The article, Thomas et al. (2006c), in particular, contains a wealth of detail on the delayed and probabilistic nature of the hazard from nuclear radiation, based on Marshall et al. (1983). Interestingly neither Jones-Lee and Chilton nor Spackman before them make reference to the very important case of assessing nuclear safety, as considered in Thomas et al. (2006c), all the more surprising in Spackman's case because his report was commissioned in the context of nuclear safety, in fact by the Nuclear Division of the Health and Safety Executive, the predecessor of today's Office of Nuclear Regulation.

It is, of course, nonsense to suggest that the use of the J-value implies a belief that the individual knows in advance the date on which he/she will die, either with or without the benefit of the safety system. On the contrary, the length of life for an individual is treated as a random variable, by definition not known in advance. An accurate assessment of change in life expectancy is however possible, both for individuals sharing the same age and for the population under consideration. The calculations make use of life tables such as those provided, by the Office of National Statistics (2017) for the UK. Such tasks are, of course, performed routinely by actuaries in the insurance industry. It is the change in life expectancy for the group of people to be protected that is used in calculating the J-value.

The ancillary suggestion of Jones-Lee and Chilton that the life expectancy gained from a safety measure will be added on at the end of life is naive. What actually happens when a protection system is implemented (with many years of service life such as the 25 years of the example of Appendix D), is that people of all ages in the protected population will tend to live longer, so that there will be rather more survivors at every age. Equation (2) above governs this effect.

Although neither Jones-Lee and Chilton nor Spackman make reference to the important topic of averting dose to reduce the number of people dying from radiation cancer, it is worth spending a little time on Lord Marshall's model (Marshall et al., 1983) for the risk of dying from exposure to ionising radiation, as it brings out clearly the effect of a delayed, stochastic hazard.

Considering first the case of a point exposure to non-acute, low-dose radiation, there will be no increase in the risk of dying from a radiation induced cancer for a decade, but the probability density will then rise to a uniform level that depends on the magnitude of the dose. The elevated risk will persist for the next 30 years before falling back to zero thereafter. Thomas et al. (2006c) explain the way that this maps onto increased hazard rates over longer periods for more complex cases, where the exposure to ionizing radiation is prolonged. The change of life expectancy for a subset of the population potentially exposed will depend on the whole history of the changed hazard rates over decades.

The Marshall model may be used to examine further Jones-Lee's and Chilton's first sentence of the last paragraph in their Section 3 (part of which has been considered in Section 2.4 above):

"In short, the fundamental conceptual limitation of the J-value model is that it focuses exclusively on the increase in life expectancy resulting from a reduction in the hazard rate for any given year and, as a result, is incapable of taking

account of the very much higher degree of fear or dread with which most people view the prospect of immediate (or very early) death relative to premature death in later years of life."

and to illustrate the lack of understanding on their part that may have given rise to such a statement.

The first thing to note, of course, is that the change in life expectancy must depend on the profile of the radiation dose over time. Indeed, the radiation may well persist for many decades, as analysed after the big nuclear accidents at Chernobyl and Fukushima Daiichi (Waddington et al., 2017). It would be quite wrong to attempt such a calculation with the hazard rate altered only for one year, as possibly recommended by Jones-Lee and Chilton. The espousal of such an idea would go against the precepts of physics, biology and actuarial science.

The next thing to observe is that, as the risk of death from a radiation-induced cancer is delayed for at least 10 years, the hazard from a continuing dose will remain at an elevated level for years longer than the duration of the dose. In fact, careful mathematical analysis allows the average age at death to be determined for those unfortunate enough to be victims of a radiation-induced cancer, both for any given starting age and for a population as a whole. This has been done for both a point exposure and for a uniform dose over a finite period of time (Thomas, 2017c). It is found that the average radiation cancer victim in the UK will live into his or her sixties or seventies, and that the average loss of life expectancy of radiation cancer victims will lie between 9 and 22 years, roughly half or less of the loss of life expectancy, 42 years, lost by the average person undergoing an immediately fatal accident in a train crash or a car crash. This makes it clear, *inter alia*, that the UK VPF is unsuitable for assessing how much to spend on reducing the risk of a nuclear accident. This would be so even if the figure assigned to the VPF had a basis in evidence, which, as previously noted, the UK VPF has not.

Interestingly, Jones-Lee and Chilton now seem to concede the point by citing, at the end of paragraph 4 of their Section 1, their recent paper on relative (different) valuations of cancer deaths versus car crash fatalities (McDonald et al., 2016). The paper by McDonald et al. suggests that people will wish to spend significantly less to avert a cancer that is delayed by decades. In this study, 157 citizens of Newcastle upon Tyne between the ages of 30 and 50 were asked to identify indifference probabilities for instantaneous death on the road in the year after next and for death from cancer after 25 years. 136 people produced answers that could be processed to give a ratio of the VPF associated with the delayed cancer to the VPF of immediate death in a car crash in 2 years' time. The geometric mean of the 136 estimates of $VPF_{\text{cancer}}/VPF_{\text{car crash}}$ was 0.228, subject a 95% confidence interval of 0.095 to 0.545.

In fact, the use of the geometric mean as a consolidation measure violates the criterion of Structural View Independence. The geometric mean will always be less than the arithmetic mean that ought to be used when interpreting people's opinions if unanswerable claims of bias are to be avoided (Thomas, 2014). Nevertheless the McDonald paper is of general interest because the cancer delay of 25 years coincides with the mean delay before death of a radiation-induced cancer under the Marshall model. Moreover, from Thomas (2017c), the ratio of what a radiation cancer victim

will lose, on average, compared with his/her loss in an immediately fatal rail accident or car crash lies between 0.214 and 0.524. This prompts the thought that what McDonald, Chilton, Jones-Lee and Metcalf were observing (imperfectly because of the bias introduced by their choice of the geometric mean) was a sample of the people in Newcastle valuing the cancer risk in proportion to their loss of life expectancy rather than according to some vaguer notion of dread.

The McDonald paper (of which Jones-Lee and Chilton were co-authors) would then imply that people do not want to die early, whether from a cancer or in a road crash, and weigh up their potential loss of life expectancy in the way modelled by the J-value. That people's judgements on life extension should conform with the J-value would not, of course, be surprising, as it has been shown that the J-value describes well the way people all over the world decide on how much to spend on life extending measures (Thomas and Waddington, 2017a).

2.6 Fallacy 6: that the risk-aversion used in J-value analysis has remained at its 2006 estimate

As explained in Appendices A and B, risk-aversion, ε , is one of the parameters used in the LQI and in J-value analysis (see Thomas (2016) for a definition of risk-aversion and the history of its use since 1728). In 2006, Thomas et al. (2006a) followed Pandey and Nathwani (2003) in their derivation of a value for risk-aversion based on an estimate of the average fraction of time spent working via the "work-life balance" (see equations (A.2), (A.4) and (A.5) of Appendix A).

Jones-Lee and Chilton devote the whole of their Section 2, plus their end-note 5, to a parallel confirmation of the J-value as presented in Thomas et al. (2006a). They claim their way to be more direct (although apparently it fails to derive a key result concerning the optimal fraction of time spent working). They then assert in paragraph 1 of their Section 4 that

"the income-leisure choice model underpinning Thomas et al.'s (2006) analysis is somewhat oversimplified and unrealistic."

and spend most of the rest of that paragraph criticising the accuracy of that model. In fact, that model and the method of estimating risk-aversion were upgraded soon after Thomas et al. (2006a) appeared.

Pandey, Nathwani and Lind produced an improved economic model in 2006 (Pandey et al., 2006) and this led them to modify their estimate of risk-aversion. This economic model was adopted in Thomas et al. (2010). The latter, 2010 paper also derived a more accurate figure for the UK working time fraction based on statistics from the Office of National Statistics.

Further progress followed in 2017 and the latest figure for risk-aversion used in J-value calculations has been measured empirically from pan-national data (Thomas and Waddington, 2017a). The new value for risk-aversion has also been reconciled with the 2010 figure on the basis of 50:50 deal being struck between the employer and the employee on how much satisfaction the average employee will gain from his work.

It follows that Jones-Lee's and Chilton's comments on the accuracy of the 2006 figure for work-life balance and hence risk-aversion ceased to have relevance after 2010 and are similarly irrelevant today.

2.7 Fallacy 7: that the discount rates used in J-value analysis have remained at their 2006 estimates

Remarks similar to those made in Section 2.6 apply here too. Jones-Lee and Chilton spend most of paragraph 1 of their Section 5 considering a range of discount rates and are critical the discount rates employed in Thomas et al. (2006a).

These criticisms are irrelevant as the rates now used with the J-value are based on empirical evidence from pan-national data (Thomas and Waddington, 2017a).

The principle of the J-value has, of course, stayed unchanged: this subsection and its predecessor have been concerned only with the problem of finding the best values of the parameters with which to populate the J-value model.

It is worth adding that the some of the data to be used in calculating the J-value, such as life tables and GDP per head, will also vary with the nation in question. These data may also change over time, making it necessary to match both the actuarial and the economic figures to the date at which the J-value is needed. Often that will mean using the most up-to-date information, but not in the case of a retrospective calculation such as estimating J-values for the Chernobyl nuclear accident a quarter of a century after the event (e.g. Waddington et al., 2017).

2.8 Fallacy 8: that the J-value should be rejected as too simple

Jones-Lee and Chilton suggest in the final paragraph of their Section 1 that the J-value is "too simplistic" and complain in the opening paragraph of their Section 3 that:

"Prima facie, the J-value model would therefore appear to provide a simple and direct means of estimating a willingness to pay-based VPF without the need to carry out a relatively complicated and demanding stated preference or revealed preference study"

A jarring feature of this statement is its attempt to lump together stated preference and revealed preference method, even though they are diametrically opposed methods.

Revealed preferences correspond to John Locke's dictum (Locke, 1690):

"I have always thought the actions of men the best interpreters of their thoughts"

The J-value is a revealed preference method, which has been validated against pan-national data (Thomas and Waddington, 2017a).

By contrast, stated preference techniques, of which Jones-Lee and Chilton are proponents, suffer from the significant defect identified by Fujiwara and Campbell (2011):

"respondents in stated preference surveys may have an incentive to deliberately misrepresent their true preferences in order to achieve a more desirable outcome for themselves ... individuals may overstate their valuations of the good if they believe their responses influence its provision and are unrelated to the price they will be charged for it"

Jones-Lee and Chilton were, of course, co-authors of the stated preference study on which the UK VPF is based. Interestingly, the most severe problems surrounding that study and hence the UK VPF came not so much from the problems raised by Fujiwara and Campbell (although a degree of exaggeration may well have been present in their first study (Beattie et al., 1998), the results of which they dismissed as "aberrant", but which were, in fact, fully understandable (Thomas and Vaughan, 2014)). No; the problems arose from the introduction of a complicated way of interpreting their opinion survey results, namely the two-injury chained method, which their own data showed was invalid, as discussed in Section 2.3 above.

Jones-Lee and Chilton then make their final charge against the J-value in the closing words of their last paragraph: "this is a gross over-simplification". Such a claim merits further investigation.

Complaining about a theory because it is simple is unusual both in science and in scientific philosophy. Scientists normally adopt Ockham's razor and prefer the simpler explanation. Here an interesting example is provided by the mathematician, John L. Casti (1991):

"The commonly held view is that Copernicus's heliocentric model vanquished the competition, especially the geocentric view of Ptolemy, because it gave better predictions of the positions of the celestial bodies. In actual fact, the predictions of the Copernican model were a little worse than those obtained using the complicated series of epicycles and other curves constituting the Ptolemaic scheme, at least to within the accuracy available using the measuring instruments of the time. No, the real selling point of the Copernican model was that it was much *simpler* than the competition, yet still gave a reasonably good account of the observational evidence."

Meanwhile, the philosopher, Sir Karl Popper (1934) explains why simplicity is so highly prized in the following terms:

"Simple statements, if knowledge is our object, are to be prized more highly than less simple ones *because they tell us more; because their empirical content is greater; and because they are better testable.*"

(Popper's italics.) He later suggests (Popper, 1983) that

"the following maxim holds for all sciences: Never aim for more precision than is required for the problem in hand."

This has echoes of Einstein's earlier words (Einstein, 1934):

"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."

now often relayed in the compressed version: "Everything should be made as simple as possible but not simpler" (Calaprice, 2011). Einstein (1934) makes his support for the simplest theories further clear:

"It is essential for our point of view that we can arrive at these constructions and the laws relating them one with another by adhering to the principle of searching for the mathematically simplest concepts and their connections."

So simplicity is regarded by scientists and philosophers as a good thing in a scientific theory rather than the disadvantage painted by Jones-Lee and Chilton. And the J-value may indeed be regarded as scientific in the terms laid down by Sir Karl Popper (1934), as it can be tested against empirical evidence (Thomas and Waddington, 2017a, Thomas, 2017a).

But how simple is the J-value? The hardest task in applying the J-value is to calculate the change of life expectancy, which requires first a knowledge of the life tables for the country concerned and then the expertise to carry out the calculation. In fact, life tables applicable to 180 nations were assembled as part of the NREFS project (Thomas and May, 2017), together with the necessary economic data, so that the J-value can already be applied in over 90% of the world's sovereign states. The calculational methods are relatively straightforward for cases where the hazard to be averted is an immediately fatal accident, although more complex where death, should it occur, is delayed and related by a stochastic process to a prior toxic exposure. Examples of the latter are asbestosis, pneumoconiosis and radiation-induced cancer. Nevertheless, methods have been developed for such cases, which can cope with a time-varying exposure lasting a hundred years or more.

Turning to the theory, the J-value draws heavily on the LQI, an elegant and powerful construct put forward by Nathwani and Lind (1997). The recent validation exercise (Thomas and Waddington, 2017a) has removed the need to consider discounting future life expectancy and thus allowed a further simplification, as discussed in Appendix A. Moreover, the same validation exercise allowed risk-aversion to be specified for developed and developing nations, thus obviating the need to relate it to the work-life balance (although a route for doing so is also given). The additional theory behind the J-value is then laid out in Appendix B.

Simplicity is a difficult thing to quantify, but it is hoped that the J-value, as applied to real-world situations, preserves the powerful simplicity of the LQI on which it is based. However, the elegance of the LQI should not disguise from us the fact that it deals with a near-infinite multiplicity of complicated things: nothing less than all the desirable activities that human beings choose to do, conditioned, as must be the case, by their resources.

The utility of income may be written in the form, $u(G) = G^{1-\varepsilon}$, and this fact is allowed for by the LQI, Q , as given in equation (A.5), repeated below:

$$Q = G^{1-\varepsilon} X \quad (\text{A.5})$$

This reveals the LQI to be the sum total of the future utility for the average person over the rest of his/her life. Thus the LQI can be seen to be a very general statement; moreover, the evidence shows that the J-value is applicable to all societies in the world for which data exist, as demonstrated in Thomas and Waddington (2017a).

A strikingly similar description of the value of life is proposed from a philosopher's perspective by John Broome (2006):

"First of all, the value of a person's life is a quantitative matter. Some people's lives go better than those of others - some people have better lives than others. One of the things that helps to determine how good your life is, is how long it goes on for. For most of us, having a longer life would be better than having a shorter life. If you expose somebody to risk by driving down a street and, as a matter of bad luck, you kill her, then what you have done is to deprive her of the rest of her life. That may be a big deprivation or it may be a comparatively small deprivation. If she is young, with a full life ahead of her, it is a very big harm that you have imposed on her. On the other hand, if she was not very far from death in any case, then it is a much smaller harm that you have imposed on her. The value of life is a quantitative matter. Some people suggest that life has an infinite value but it does not: it has a finite value and it is a matter of quantity. That is one point.

"The second point I would like to make is that the value of a person's life is comparable, or commensurable as philosophers often say, with the value of mundane things such as chatting to your friends or having a holiday. ... what you lose, if you lose your life, is the sort of mundane goods that you have in your life. If your life is shortened by a year, what you lose is your annual holiday and all the fun you would have during that year – the chatting to friends, and so on, that you would have during that year. In fact, the value of life is nothing other than the value of the mundane goods that occur in it and so there is no problem with commensurability between life and other goods, and they can be measured on the same scale."

It would be impossible, as a practical possibility, to enumerate each of the "mundane goods" in each person's life, let alone evaluate each one in a way that mimicked the approach of the person benefiting. The problem can be circumvented only by postulating a very general measure of the quality of life, namely the LQI, which, as we have seen, is the sum of the average person's utility from now on.

To explore the issues involved in any attempt to delve further into the ways in which utility is experienced, let us compare and contrast the LQI with the Human Development Index (HDI), used by the United Nations (2018a) to assess the development of a country. The HDI bears some marked similarities to the LQI, in that it includes as constituents two variables that approximate those used in the LQI, namely

- utility of Gross National Income (GNI) per capita, based on a similar value for risk-aversion, namely $\varepsilon = 1.0$ (cf. the utility of GDP per capita for the LQI,

calculated using $\varepsilon = 0.95$ when all countries in the world are considered (Thomas and Waddington, 2017a)).

- life expectancy at birth (cf. life expectancy averaged over all ages for the LQI)

However, it differs from the LQI by

- including, as an additional constituent variable, the average of the mean years of schooling that have been received by the existing adult population and the expected years of schooling for children just starting school.
- setting targets for each of its constituent variables, where the targets are generally chosen to be somewhat higher than the values pertaining in highly developed countries:
 - the utility of 75,000 International Dollars per year income
 - 85 years life expectancy at birth (combined genders)
 - 16.5 years of average schooling (including higher education)
- calculating the relative distance of each of its 3 constituent variables from the target provided
- calculating the final HDI from the third root of the product of the three relative distances.

It will be seen immediately that the HDI, although sharing a number of similarities with the LQI, is less simple and more prescriptive. "Higher marks" are given the closer the nation under examination approaches the practice of highly developed nations. In particular the HDI will "reward" with a higher index a nation that chooses to devote more of its resource to schooling. This makes it less general and could restrict its usefulness as a guide to modifying national policy and behaviour. After all, it might well be that the nation could achieve higher overall welfare by devoting more of its resources to other, higher-priority items before striving to match the amount of schooling provided in highly developed countries.

To be fair, the HDI makes no claim to do other than provide a "summary measure of [a nation's] achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living" (UN, 2018b). But while there is no dispute that good provision of primary, secondary and tertiary education is a feature of developed countries, the prominence given to "access to knowledge" in the HDI will tend to reduce the policy choices available to the citizens of a less developed country that takes the index as its sole guide. A less constrained path to development is explored in Thomas and Waddington (2017a). The J-value model presented there (based, of course, on the LQI), offers the interpretation that the citizens of a country will continually trade off a fraction of their income to promote their longer life, and that wealth and longevity will thereby increase in tandem following the curve documented by Preston (1975). Thus the J-value model explains the route by which all countries may progress further up the Preston curve as they get richer.

Moreover, although the HDI is less simple than the LQI, it does not come close to satisfying the need identified by John Broome to represent the multiplicity of mundane things that make up the bright spots in people's lives, a sample of which he listed. But would the situation be helped by further augmentation of such an index to cover other plausible, desirable items such as the use of a gymnasium or a swimming pool, listening to music, going to the opera, attending the theatre etc? How would we

decide what to include and what to leave out for people in different nations in the world? In reality, the choice of what to do with his/her resources must be and will be left to the individual, an idea that corresponds to finding the overall utility of income but not to specifying precisely the activities in which that utility will be manifested. Thus we are directed back towards the LQI and the J-value for guidance on life extending decisions.

Finally on the interesting point of the simplicity or otherwise of a theory in the social sciences, Milton Friedman (1953,1966) comments

"A hypothesis is important if it 'explains' much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions; it takes account of, and accounts for, none of the many other attendant circumstances, since its very success shows them to be irrelevant for the phenomena to be explained.

"To put this point less paradoxically, the relevant question to ask about the 'assumptions' of a theory is not whether they are descriptively 'realistic', for they never are, but whether they are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions."

The success in the test of the J-value model and hence the LQI concept against pan-national data (Thomas and Waddington, 2017a), a non-trivial trial involving the behaviour of people in the overwhelming majority of nations in the world, shows that the J-value provides a good description of the link between changes in life expectancy and changes in average income (GDP per head). Further corroboration is offered by Thomas (2017a). It is hoped that the J-value retains sufficient simplicity to be seen as "explaining much by little" in the way advocated by Milton Friedman.

3. Discussion

It is a shame that Jones-Lee and Chilton waited for over 10 years before responding to the first J-value paper (Thomas et al., 2006a), particularly as they evince no awareness of the J-value developments that have taken place in the intervening years (nor even of a companion J-value paper, Thomas et al. (2006c), published in the same journal issue as the first). It is also a pity that they did not choose to respond directly to *PSEP*, the journal that carried that first J-value article. However we have attempted to rectify both these deficiencies, first by giving an account of how the J-value has advanced since 2006 (Appendix A) and then by submitting our response to *PSEP* for publication. Moreover, while we have worked through an example relevant to the delayed hazards of nuclear radiation in previous papers (e.g. Thomas and Jones, 2010, Waddington et al., 2017), we have included in this paper a worked example for a high hazard process plant where the risk is of death immediately or in the short term following an accident (Appendix D). We have also given details, previously unpublished, of how to calculate the gain in life expectancy from averting such an immediate risk (Appendices E and F).

While the quality of the points made by Jones-Lee and Chilton may not be high, nevertheless the process of providing answers allows some interesting implications of the J-value to be brought out. These are likely to be of interest both to users of the J-value method and more generally. Some of the more salient issues will be discussed further below.

The "principal" proposition put forward by Jones-Lee and Chilton, namely that the J-value should be regarded as invalid if less extension of life is gained from lowering the hazard rate now than from a delayed hazard reduction that is tens of times higher, has been shown to be ill thought out and wrong. Even after allowing for the extreme and unjustified weighting given by Jones-Lee and Chilton to the latter risk reduction over the former, the early lowering of hazard is shown by the J-value to be preferable by a large margin, contrary to their claims. Jones-Lee and Chilton perpetrated the economic solecism of ignoring the cost of the risk reduction.

In fact, Jones-Lee and Chilton make no attempt to substantiate their implicit assertion that a 47 times bigger reduction in risk (39 times using recent ONS life tables) in 40 years' time ought to be regarded as comparable with a much smaller hazard reduction now. Even though, as shown, such an attempt to boost the attraction of the delayed risk reduction must ultimately fail, it is self-evidently a ludicrous proposition. It is quite out of line with the action of an industrial protection system on a high-hazard process plant, which will offer the same degree of hazard rate reduction to all members of the public living nearby, irrespective of age.

A point of general interest arises nevertheless, in that the J-value confirms that it will be better to institute protection against a hazard now rather than to delay it. This finding is hardly surprising, but the J-value provides a way of quantifying the effect.

A more interesting issue is explored in Section 2.2, namely how far the J-value should apply to individuals within a group to be protected as opposed to the group as a whole. Were this to be so, the J-value for the group should be based on the average of all the individual J-values. Individual J-values allowing for age-dictated variations in life expectancy and change of life expectancy have been calculated for the worked example given in Appendix D, where a protection system with a 25 year life is under consideration for a high-hazard process plant. It is found that the individual J-values stay close to the group J-value for the middle range of ages, but start to rise at either end, both for older people and for those yet to be born at the time of installation. The reason in both cases is that these people would not experience the full duration of the protection offered by the safety system, either because they are likely to die before the system comes out of service (old people) or because its working lifetime ends too early for them (those born during its period of service). The overall average of age-dependent J-values then comes out as higher than the group J-value, but not greatly so.

But Section 2.2 shows that only by calculating the J-value for the group as a whole, using group-averaged internal values, can the proposition advanced by Sunstein (2003) be fulfilled, namely that a day of life extension is valued the same for each member of the population, rich or poor, young or old. Thus ethics demand that the J-value be calculated using average values for income, life expectancy and change of life expectancy for the group as a whole. This moral stance has now been confirmed

by the validation of the J-value against pan-national data: the J-value models the way that decisions on life extending measures are made on average in 180 out of the 193 nations in the UN. This confirms the J-value as a yardstick by which any particular decision on safety spending can be judged.

As shown in Section 2.8, Jones-Lee and Chilton find themselves in opposition to some of the world's most eminent scientists, economists and philosophers when they argue in favour of complication and against simplicity for a scientific theory such as the J-value. Their stance may, of course, be conditioned by the fact that they are originators and proponents of the UK VPF, which is based on their "relatively complicated and demanding stated preference study".

Throughout their paper, Jones-Lee and Chilton attempt to talk up the VPF and denigrate the J-value. But a major problem with their position is that their method of interpreting their stated preference data, complicated or not, has been shown to be scientifically invalid. The result is that the UK VPF has no basis in evidence. But extensive evidence now exists for the J-value being not only a simple but a valid description of the way that people value human life across the world.

4. Conclusions

A detailed examination of the criticisms of the J-value made by Jones-Lee and Chilton reveals them to be without merit. In particular, their "principal" objection has been shown to be poorly thought out and based, *inter alia*, on an elementary economic error. Their other points fare little better when subjected to scrutiny.

However, the exercise of answering the points raised by Jones-Lee and Chilton has brought out additional implications of the J-value that will be of interest both to users of the J-value method and more generally. Moreover, the task has prompted a review of J-value developments over the 12 years since the first paper appeared in *PSEP*. In addition, a worked example has been provided for a high-hazard process plant where the risk involves death either immediately or in the short term.

A feature of their critique is the attempt by Jones-Lee and Chilton, originators and proponents of the UK VPF, to talk up the VPF while talking the J-value down. However, the method used by Jones-Lee and Chilton to derive the UK VPF has been shown previously in *PSEP* to be invalid, so that the UK VPF has no basis in evidence.

By contrast the J-value has been validated against empirical data. The J-value covers both systems designed to reduce the chance of death in the short term and those intended to safeguard against death that, if it occurs, will be delayed by years or decades. In the latter case, it has been used to examine the high-profile policy of evacuation after a big nuclear accident.

The J-value may be used to assess safety measures in both high hazard and all other industries. A fully objective and validated method is thus currently available to answer the vexed question of how much ought to be spent on protecting human life.

Appendix A. The evolution of the life quality index and J-value since 2006

The LQI, Q , recommended by Pandey and Nathwani (2003), had the form

$$Q = G^q X_d \quad (\text{A.1})$$

where G was the average income, taken for ethical reasons to be Gross Domestic Product (GDP) per person, and X_d was the population-average discounted life expectancy. (Future utility of income was the true subject of the discounting, but this is equivalent to discounting life expectancy, as explained in Thomas et al., 2010, Section 5). Meanwhile q was the ratio of time spent in work to that spent not working, in other words, the "work-life balance":

$$q = \frac{w_0}{1 - w_0} \quad (\text{A.2})$$

in which w_0 is the population-average time spent working. The formulation of Equation (A.1) was used in the first J-value paper (Thomas et al., 2006a), which also gave a general derivation for the LQI based on the postulate that the fundamental variables influencing the current quality of life for the average individual were first how long he/she could expect to live from now on and secondly how much he/she would have available to spend, both on life's necessities and on its luxuries.

Pandey et al. (2006) then introduced a Cobb-Douglas Production Function to model the use of labour in national production, leading to a revised expression for q :

$$q = \frac{1}{\theta} \frac{w_0}{1 - w_0} \quad (\text{A.3})$$

with θ equal to the share of wages in GDP. The newer expression for q was adopted for the J-value in Thomas et al. (2010), where it was also shown that

$$q = 1 - \varepsilon \quad (\text{A.4})$$

in which ε is risk-aversion (or the negative of the elasticity of marginal utility of income).

The general form of the LQI (equations (A.1) and (A.4)) was used in 2017 (Thomas and Waddington, 2017a) to model the decisions of those considering the adoption of life-extending measures all over the world, assuming that their decisions were governed by a J-value of 1.0. Good correspondence between actual and predicted population-average life expectancy was found for 180 out of the 193 nations recognised by the United Nations in 2009 and excellent correspondence for 162 countries.

Based on empirical data, the value of risk-aversion, ε , used in taking life extending decisions was found to be 0.91 for developed countries, close in value to previous, independent estimates. Moreover, it was possible to reconcile the new figure with the

2010 value on the basis of a 50:50 bargain between employer and employee on the amount of personal satisfaction the employee derives from his work. In addition, the best J-value model resulted from setting the rate used to discount future life expectancy, the "net discount" rate, to zero. This implies a very low value of pure time discount rate, a result that conforms closely with the work of Ramsey (1928) and Stern (2007, 2009, 2015), the former going so far as to describe the practice of discounting later enjoyments as "ethically indefensible" and arising "merely from the weakness of the imagination".

The validation exercises (Thomas and Waddington, 2017a, Thomas, 2017a) have thus established the LQI for use with the J-value as:

$$Q = G^{1-\varepsilon} X \quad (\text{A.5})$$

where population-average values are used for income, G , and life expectancy, X .

While the previous work on the LQI, particularly the fundamentally important insights of Nathwani and Lind (1997), guided the choice of function, *viz.* $G^{1-\varepsilon} X$, the empirical confirmation of the validity of equation (A.5) means that its stands independent of the arguments used to derive the LQI.

Since the utility of income may be written in the form $u(G) = G^{1-\varepsilon}$ (Thomas, 2016), equation (A.5) yields the intuitively satisfying interpretation that the life quality of the average person in the group is the sum of the future utility of his/her income over the expected future years. This interpretation is strongly in line with philosopher John Broome's perspective (2006). See also Section 2.8.

Appendix B. Finding the J-value

B.1. Deriving the J-value from the LQI

Perturbing equation (A.5) yields:

$$\frac{\delta Q}{Q} = (1-\varepsilon) \frac{\delta G}{G} + \frac{\delta X}{X} \quad (\text{B.1})$$

where δX is the average change in life expectancy across the population. The requirement that the safety measure should not produce a net disbenefit is that $\delta Q \geq 0$, with the limiting condition, $\delta Q = 0$. Applying this to equation (B.1) gives the maximum it is worth paying per person per year to achieve a gain in population-average life expectancy, δX , as:

$$\delta G = -\frac{G}{1-\varepsilon} \frac{\delta X}{X} \quad (\text{B.2})$$

where the negative sign preceding the terms on the right indicates a reduction in effective annual income. The J-value is then the ratio of the actual cost per year, $\delta \hat{G}$, to the maximum reasonable cost per year:

$$J = \frac{\delta \hat{G}}{\delta G} \quad (\text{B.3})$$

Hence the J-value may be written:

$$J = \frac{1-\varepsilon}{G} \delta \hat{G} \frac{X}{\delta X} \quad (\text{B.4})$$

$J > 1$ implies that the safety measure is, in the absence of special pleading to the contrary, too expensive.

B.2. Converting actual up-front and ongoing payments into a single annual payment, $\delta \hat{G}$

In the general case, a safety system may involve the following costs:

- an up-front expenditure, $\delta \hat{V}_{N1}$,
- a stream of payments, $\delta \hat{\gamma}_N(t)$, consisting of interest and some repaid principal, made over a period of y years at a commercial interest rate net of inflation, r_c . [The stipulation that this rate needs to be net of inflation means that r_c is likely to be close to the social discount rate, r^* , which is found to be equal to the long-term national growth rate per head (Thomas and Waddington, 2017a, Thomas, 2017a)].
- the cost of maintenance, $\delta \hat{m}(t)$, of the system over its lifetime in service, T_S .

The up-front equivalent, $\delta \hat{V}_{N2}$, of the payments, $\delta \hat{\gamma}_N(t)$, may be calculated by amortizing over the repayment period, y , using the commercial interest rate, r_c :

$$\delta \hat{V}_{N2} = \int_0^y e^{-r_c t} \delta \hat{\gamma}_N(t) dt = \delta \hat{\gamma}_N \frac{1 - e^{-r_c y}}{r_c} \quad (\text{B.5})$$

where the last step conforms to the assumption that the payment is the same each year, viz. $\delta \hat{\gamma}_N(t) = \delta \hat{\gamma}_N$ for all t in the interval considered. Similarly, the up-front equivalent of an annual maintenance cost, $\delta \hat{m}$, will be:

$$\delta \hat{V}_{N3} = \delta \hat{m} \frac{1 - e^{-r_c T_R}}{r_c} \quad (\text{B.6})$$

The total equivalent up-front cost, $\delta \hat{V}_N$, is then found by adding all the components:

$$\delta \hat{V}_N = \sum_{i=1}^3 \delta \hat{V}_{Ni} \quad (\text{B.7})$$

Making the usual J-value assumption of a steady state population, the number of people able to make a payment at any given time will always be the same, namely the number, N , of people alive at any given time, now and in the future. Hence the average total amount, $\delta\hat{V}$, notionally paid by one person is given by

$$\delta\hat{V} = \frac{\delta\hat{V}_N}{N} \quad (\text{B.8})$$

$\delta\hat{G}$ is found from $\delta\hat{V}$ by distributing this sum over an appropriate time interval using the inverse process described by equations (B.5) and (B.6) but now using the social discount rate, r^* .

Someone whose life is preserved by a safety system will enjoy the benefit, *viz.* remaining alive, for the rest of his/her life, as explained by Broome (2006). Such a benefit will be enjoyed on average over the population-average life expectancy, suggesting this length of time should be the notional repayment term.

There are two candidates for the population-average life expectancy for use in this way: (i) the mean life expectancy, X_1 , of those alive at the installation of the protective measure, and (ii) the average expected life to come, X_p , of those living and those born during the service lifetime, T_s , of the protection system, where

$X_1 \leq X_p \leq X(0)$ as $0 \leq T_s \leq \infty$, with the life expectancy at birth, $X(0) \approx 2X_1$ for developed countries (Thomas and Waddington, 2017a). To distinguish between the two options, it is useful to appeal to the notion of intergenerational equity, where the United Nations Educational Scientific and Cultural Organization concluded in its "Declaration on the Responsibilities of the Present Generations Towards Future Generations", Article 1 (UNESCO, 1997):

"The present generations have the responsibility of ensuring that the needs and interests of present and future generations are fully safeguarded."

Also of relevance is the call of the World Health Organization (WHO, 2015), in the context of health inequalities, for:

"action to stop children from inheriting health risks from their parents and grandparents and passing them on to their own offspring."

The shorter repayment period, X_1 , corresponds to the idea of strong inter-generational equity, in the sense that the notional loan will be repaid by the end of the lifetime of the average person in the current generation, taken to be those living at the time of installation. Thus the current generation will spare all succeeding cohorts of any part of the financial burden. The second conforms to a weaker form, where the repayment period will be longer, at X_p , which will be up to twice the average lifetime of those living at the time the safety measure was installed. Both options limit the length of time over which payment for the safety measure is made – the "mortgage" is not extended indefinitely into the future, in line with the need for intergenerational fairness. The stronger form means that the bill will be paid in full at the end of the

lifetime of the current generation, while the weaker alternative extends the payment period some way into the lifetime of the succeeding generation but not beyond, again limiting the financial burden on future generations.

The appropriate discount rate for calculating the notional annual repayments will be the social discount rate, r^* , which is to be equal to the long-term national growth rate per head (Thomas and Waddington, 2017a, Thomas 2017a). Hence

$$\delta\hat{G} = \frac{r^*}{1 - e^{-r^*T}} \frac{\delta\hat{V}_N}{N} \quad \text{where } T = \begin{cases} X_1 & \text{for strong inter-generational equity} \\ X_p & \text{for weak inter-generational equity} \end{cases} \quad (\text{B.9})$$

B.3. The change in population-average life expectancy conferred by the safety measure

Two possible changes in population-average life expectancy, δX_1 and δX_p , may be considered when calculating the J-value, corresponding to the two population averages discussed immediately above.

The combination $(X_1, \delta X_1)$ will give a J-value based on strong inter-generational equity, while the combination $(X_p, \delta X_p)$ will give a J-value corresponding to weak inter-generational equity. The second J-value will tend to be somewhat lower than the first. Meanwhile using the mixed combination $(X_1, \delta X_p)$ will give a J-value that is lower than the "strong" case.

Appendix C. The inappropriateness of the harmonic mean recommended by Jones-Lee and Chilton

Let A and B be jointly distributed random variables, with

$$\begin{aligned} E(A) &= \mu_A \\ E(B) &= \mu_B \end{aligned} \quad (\text{C.1})$$

The covariance of A and B is defined (e.g. Rice, 2007, Section 4.3) as the expected value of the product of the deviations of A and B from their respective means:

$$\text{cov}(A, B) = E((A - \mu_A)(B - \mu_B)) \quad (\text{C.2})$$

Expanding the right-hand side of equation (C.2) gives:

$$\begin{aligned} \text{cov}(A, B) &= E(AB - A\mu_B - B\mu_A + \mu_A\mu_B) \\ &= E(AB) - E(A)\mu_B - E(B)\mu_A + \mu_A\mu_B \\ &= E(AB) - \mu_A\mu_B - \mu_A\mu_B + \mu_A\mu_B \\ &= E(AB) - E(A)E(B) \end{aligned} \quad (\text{C.3})$$

where the distributive property of the expectation operator has been used in the second line and equation set (C.1) in the third and fourth. Hence the expectation of the product, AB , may be written as:

$$E(AB) = E(A)E(B) + \text{cov}(A, B) \quad (\text{C.4})$$

In the case where the random variable, B , in equation (C.4) is the reciprocal of a random number, C , the equation may be written:

$$E\left(\frac{A}{C}\right) = E(A)E\left(\frac{1}{C}\right) + \text{cov}\left(A, \frac{1}{C}\right) \quad (\text{C.5})$$

Jones-Lee and Chilton effectively argue that the term, $E(1/C)$, in equation (C.5) can be replaced by $1/H$, where H is the harmonic mean. This is not legitimate, as the reciprocal of the harmonic mean will not generally coincide with the expected value, $E(1/C)$.

This is because the harmonic mean is defined only for a finite number, n , of discrete, strictly positive values (Rade and Westergren, 2004), when the reciprocal of H is the arithmetic mean of the reciprocals, $1/c_i$, $i = 1, 2, \dots, n$:

$$\frac{1}{H} = \frac{1}{n} \left(\frac{1}{c_1} + \frac{1}{c_2} + \dots + \frac{1}{c_n} \right) \quad (\text{C.6})$$

It is clear from equation (C.6) that the reciprocal of the harmonic mean may be regarded as the expected value, $E(1/C)$, only in the restricted circumstance that the random variable, C , is governed by a discrete uniform distribution. This could not be the case in the current situation where the denominator of the ratio, *viz.* C , would be a function of age, implying that uniformity could not apply. Put simply, there are far fewer people aged 95 than there are aged 25. It would be necessary to use an empirical probability distribution to account for the differing fractions of people at each age.

Hence the statement by Jones-Lee and Chilton:

"Furthermore, in the case of a ratio, even if the numerator and inverse of the denominator are uncorrelated, it is the *harmonic* mean of the denominator – rather than the arithmetic mean – that is required in order to compute the arithmetic mean of the ratio." (their italics)

is incorrect.

Appendix D. Worked example: process plant possessing a major accident hazard

Suppose that a chemical plant is to be built close to a small town with a steady population of 10,000 citizens. The process plant is assumed to fall into the COMAH

(Control of Major Accident Hazard) category and it is estimated that 100 people in this township would be killed in the eventuality of a severe accident occurring. Such accidents would be rare, but are estimated to occur with a frequency of once in a thousand years, implying a probability of occurrence in any given year of about 10^{-3} . This imposes an additional hazard rate on each person in the community near the plant of $10^{-3} \times 100 \div 10,000 = 10^{-5}$. The plant operator wishes to reduce the frequency of the accident. He can reduce the accident frequency by a factor of 100 by installing a protection system that will last for 25 years and has a cost at installation of £9M and an annual maintenance cost of £30,000. The commercial interest rate to which the operator has access is 3% per annum real, that is to say net of inflation. Is this expenditure reasonable?

Discounting the maintenance costs at 3% p.a. over the service lifetime of 25 years using equation (B.6) gives $\delta \hat{V}_{N3} = £527,633$, so that the total equivalent cost of the protection system at installation is £9,527,633, which corresponds to £953 per person. Spreading payments over the average life expectancy, 41.64 years, of those living at the time of installation, gives an expenditure, $\delta \hat{G} = £33.90$ per year. Here the long-term growth of UK GDP per head, 2.03 % p.a. (World Bank, 2017), is used as the social discount rate. Hence the annual payment of £33.90 corresponds to the assumption of strong intergenerational equity. The figure reduces slightly to £30.02 per year when the life expectancy used, 50.9 years, corresponds to a population that includes those being born during the service period of the protection system in addition to those living at the time of installation – the condition of weak intergenerational equity.

The actuarial calculations are carried out by perturbing the hazard rate for people of all ages comprising a subset of the UK population based on the latest life tables available, using data from 2014 to 2016 (Office of National Statistics, 2017). The resulting change to the hazard rate, which is significantly lower than the natural hazard rate, is imposed for the next 25 years of life on people of all ages. The change in life expectancy is 2.75 days for those living at the time of installation, 3.28 days for the cohort born after installation and before the end of the service lifetime and 2.88 days for the combined group.

Risk-aversion is given by $\varepsilon = 0.91$, as found by Thomas and Waddington (2017a). Meanwhile GDP per head in 2015 is $G = £ 29,008$ in 2015 £s (Office of National Statistics, 2018).

J-values are produced shown in Table 2. These show a small variation, between 0.51 and 0.58, depending on the assumption made on intergenerational equity. In all cases, however, $J \leq 1$ and so the protection system represents good value for money.

Appendix E. Increase in life expectancy when the hazard rate is reduced by a uniform amount for longer than one year: those living at the time of installation

An expression is derived here for the change in life expectancy for those living at the time of installation of the protection measure.

Assume that a protection system with a service lifetime, T_s , is implemented that reduces the hazard rate by a uniform amount, b , for the duration of the time it is in service. Consider the people to be protected who are living at the time the safety measure is installed. Assuming that there is an ultimate maximum age, α_0 , to which people may live, the period of benefit, $T(a)$, from the safety system for someone of age, a , will be limited to

$$T(a) = \min(T_s, \alpha_0 - a) \quad (\text{E.1})$$

The reduction in hazard rate, $b(u)$, at current and future ages, u , that people of starting age, a , will experience is thus

$$b(u) = \begin{cases} b & \text{for } a < u \leq a + T(a) \\ 0 & \text{for } u > a + T(a) \end{cases} \quad a \geq 0 \quad (\text{E.2})$$

Before the safety system has been installed, the survival probability, $S(t|a)$, to age, t , for someone of age, a , is

$$S(t|a) = \frac{S(t)}{S(a)} = e^{-\int_{u=a}^t h(u) du} \quad (\text{E.3})$$

(see e.g. Thomas et al., 2006c, Appendix 1). After the safety system has been installed, the new survival probability, $S'(t|a)$, conditional on a , will be

$$\begin{aligned} S'(t|a) &= e^{-\int_{u=a}^t (h(u) - b(u)) du} = e^{\int_{u=a}^t b(u) du - \int_{u=a}^t h(u) du} \\ &= e^{\int_{u=a}^t b(u) du} S(t|a) = e^{\int_{u=a}^t b(u) du} \frac{S(t)}{S(a)} \end{aligned} \quad (\text{E.4})$$

The new conditional survival probability may be used to find the life expectancy at age, a , after the implementation of the protection measure as:

$$X'(a) = \int_{t=a}^{\infty} S'(t|a) dt = \int_{t=a}^{\alpha_0} S'(t|a) dt + \int_{t=\alpha_0}^{\infty} 0 dt = \int_{t=a}^{\alpha_0} S'(t|a) dt \quad (\text{E.5})$$

where the last step follows the assumption that no-one will survive past the ultimate maximum, age, α_0 . Substituting from equation (E.4) into equation (E.5) gives:

$$X'(a) = \int_{t=a}^{\alpha_0} e^{\int_{u=a}^t b(u) du} \frac{S(t)}{S(a)} dt \quad (\text{E.6})$$

or

$$X'(a) = \frac{1}{S(a)} \int_{t=a}^{\alpha_0} e^{\int_{u=a}^t b(u)du} S(t) dt \quad (E.7)$$

Considering the term, $\exp\left(\int_{u=a}^t b(u)du\right)$, noting the conditions of equation (E.2) and taking the case where the starting age, a , is low enough to satisfy $a + T_s \leq \alpha_0$, so that the full beneficial period of the safety measure is experienced, the expression will take the form

$$e^{\int_a^t b(u)du} = \begin{cases} e^{\int_a^t b(u)du} = e^{b(t-a)} & \text{for future ages, } t : t \leq a + T_s \\ e^{\int_a^{a+T_s} b(u)du + \int_{a+T_s}^t 0 du} = e^{bT_s} & \text{for future ages, } t : t > a + T_s \end{cases} \quad a + T_s \leq \alpha_0 \quad (E.8)$$

Substituting from equation (E.8) into equation (E.7) gives the modified life expectancy for those living who are young enough to receive the full benefit of the safety system:

$$X'(a) = \frac{1}{S(a)} \left(\int_{t=a}^{a+T_s} e^{b(t-a)} S(t) dt + \int_{t=a+T_s}^{\alpha_0} e^{bT_s} S(t) dt \right) \quad a + T_s \leq \alpha_0 \quad (E.9)$$

Now consider those people whose ages, a , are high enough to satisfy $a + T_s > \alpha_0$. These people are too old to experience the full benefit of the safety measure, which can benefit them not over the full service lifetime, T_s , but only for the lesser period, $\alpha_0 - a$ (at most). For these older people, the first integral on the right-hand side of equation (E.9) will have the term, T_s , replaced by $\alpha_0 - a$, so that the upper limit becomes α_0 . By the same token, the second integral will disappear. Hence the general expression for life expectancy at age, a , after the protection measure has been installed is:

$$X'(a) = \begin{cases} \frac{1}{S(a)} \left(\int_{t=a}^{a+T_s} e^{b(t-a)} S(t) dt + \int_{t=a+T_s}^{\alpha_0} e^{bT_s} S(t) dt \right) & \text{for } a \leq \alpha_0 - T_s \\ \frac{1}{S(a)} \int_{t=a}^{\alpha_0} e^{b(t-a)} S(t) dt & \text{for } a > \alpha_0 - T_s \end{cases} \quad (E.10)$$

Meanwhile the life expectancy at age, a , without the protection measure is given by:

$$X(a) = \frac{1}{S(a)} \int_{t=a}^{\alpha_0} S(t) dt \quad (\text{E.11})$$

(cf. equation (E.5)). Thus the change in life expectancy, $\delta X_1(a) = X'(a) - X(a)$, for someone of age, a , at the time of installation, is given by:

$$\delta X_1(a) = \begin{cases} \frac{1}{S(a)} \left(\int_{t=a}^{a+T_s} (e^{b(t-a)} - 1) S(t) dt + \int_{t=a+T_s}^{\alpha_0} (e^{bT_s} - 1) S(t) dt \right) & \text{for } a \leq \alpha_0 - T_s \\ \frac{1}{S(a)} \int_{t=a}^{\alpha_0} (e^{b(t-a)} - 1) S(t) dt & \text{for } a > \alpha_0 - T_s \end{cases} \quad (\text{E.12})$$

The change in life expectancy, δX_1 , averaged over all ages in the population will be:

$$\begin{aligned} \delta X_1 &= \int_{a=0}^{\alpha_0} f_A(a) \delta X(a) da \\ &= \int_{a=0}^{\alpha_0 - T_s} f_A(a) \delta X(a) da + \int_{a=\alpha_0 - T_s}^{\alpha_0} f_A(a) \delta X(a) da \end{aligned} \quad (\text{E.13})$$

where $f_A(a)$ is the probability density for age, a . For a population in the steady state:

$$f_A(a) = \frac{S(a)}{X(0)} \quad (\text{E.14})$$

(see Thomas, 2017c, Appendix B). Substituting from equations (E.12) and (E.14) into equation (E.13) gives the change in life expectancy for those living at the time of installation of the safety system:

$$\delta X_1 = \frac{1}{X(0)} \left(\int_{a=0}^{\alpha_0 - T_s} \left(\int_{t=a}^{a+T_s} (e^{b(t-a)} - 1) S(t) dt + \int_{t=a+T_s}^{\alpha_0} (e^{bT_s} - 1) S(t) dt \right) da + \int_{a=\alpha_0 - T_s}^{\alpha_0} \int_{t=a}^{\alpha_0} (e^{b(t-a)} - 1) S(t) dt da \right) \quad (\text{E.15})$$

or

$$\delta X_1 = \frac{1}{X(0)} \left(\int_{a=0}^{\alpha_0 - T_s} \int_{t=a}^{a+T_s} (e^{b(t-a)} - 1) S(t) dt da + \int_{a=0}^{\alpha_0 - T_s} \int_{t=a+T_s}^{\alpha_0} (e^{bT_s} - 1) S(t) dt da + \int_{a=\alpha_0 - T_s}^{\alpha_0} \int_{t=a}^{\alpha_0} (e^{b(t-a)} - 1) S(t) dt da \right) \quad (\text{E.16})$$

Appendix F. Increase in life expectancy when the hazard rate is reduced by a uniform amount for longer than one year: those born into the protected group during the time the safety system is in service

Let the change in life expectancy for someone born τ years after the safety measure has been installed be $\delta X^{(\tau)}(0)$. Let us call τ the "birth delay". For a safety system with a service lifetime of T_S years, someone born immediately after the safety system has been installed will potentially receive benefits for the full T_S years, beginning at the age of 0. In this case the change in life expectancy will be

$$\delta X^{(0)}(0) = \delta X(0) \quad (\text{F.1})$$

where $\delta X(0)$ is the change in life expectancy for a starting age, a , of zero:

$$\delta X(0) = \delta X(a) \Big|_{a=0}.$$

Individuals born 1 year after the safety system has been installed will accrue benefits over a period of $(T_S - 1)$ years, again starting at age 0. These will lead to a change in life expectancy, $\delta X^{(1)}(0)$. Those born 2 years after system installation will accrue benefits over $(T_S - 2)$ years, starting at age 0. They will experience a change in life expectancy, $\delta X^{(2)}(0)$. This process will continue until we consider an individual born $(T_S - 1)$ years after installation, who will have just 1 year of benefit starting at age 0. The corresponding change in life expectancy will be $\delta X^{(T_S - 1)}(0)$. Meanwhile anyone born T_S or more years after installation will experience no benefits and hence no change in life expectancy: $\delta X^{(T_S)}(0) = 0$.

Considering the cohort of the individuals to be born during the period of service of the safety system, their reduction in hazard rate, $b(u)$, at future ages, u , will be

$$b(u) = \begin{cases} b & \text{for } 0 < u \leq T_S - \tau \\ 0 & \text{for } u > T_S - \tau \end{cases} \quad (\text{F.2})$$

The general analysis laid out in Appendix E applies and, specifically, equation (E.7). For the relevant case where the starting age is $a = 0$ and the birth delay is τ

$$X^{(\tau)}(0) = \frac{1}{S(0)} \int_{t=0}^{\alpha_0} e^{\int_{u=0}^t b(u) du} S(t) dt = \int_{t=0}^{\alpha_0} e^{\int_{u=0}^t b(u) du} S(t) dt \quad (\text{F.3})$$

since $S(0) = 1$. Considering the term, $\exp\left(\int_{u=0}^t b(u) du\right)$, applying the conditions of equation set (F.2) gives

$$e^{\int_0^t b(u)du} = \begin{cases} e^{\int_0^t bdu} = e^{bt} & \text{for future ages, } t : t \leq T_S - \tau \\ e^{\int_0^{T_S-\tau} bdu + \int_{T_S-\tau}^t 0du} = e^{b(T_S-\tau)} & \text{for future ages, } t : t > T_S - \tau \end{cases} \quad (\text{F.4})$$

Partitioning the integral in equation (F.3) above and below $t = T_S - \tau$ and substituting from equation (F.4) produces:

$$X^{(\tau)}(0) = \int_{t=0}^{T_S-\tau} e^{bt} S(t) dt + \int_{t=T_S-\tau}^{\alpha_0} e^{b(T_S-\tau)} S(t) dt \quad (\text{F.5})$$

Meanwhile the equivalent life expectancy in the absence of the safety measure is:

$$X^{(\tau)}(0) = \int_{t=0}^{\alpha_0} S(t) dt = \int_{t=0}^{T_S-\tau} S(t) dt + \int_{t=T_S-\tau}^{\alpha_0} S(t) dt \quad (\text{F.6})$$

Subtracting equation (F.6) from equation (F.5) gives the change in life expectancy when the birth delay is τ :

$$\delta X^{(\tau)}(0) = \int_{t=0}^{T_S-\tau} S(t)(e^{bt} - 1) dt + \int_{t=T_S-\tau}^{\alpha_0} S(t)(e^{b(T_S-\tau)} - 1) dt \quad (\text{F.7})$$

For a steady-state population, the rate of birth will be constant, and so the probability density for birth delay, $f_T(\tau)$, will be uniform and given by:

$$f_T(\tau) = \frac{1}{T_S} \quad (\text{F.8})$$

The change in life expectancy, δX_2 , for the cohort of those born during the period of service of the protection system is then found as:

$$\delta X_2 = \frac{1}{T_S} \int_{\tau=0}^{T_S} \delta X^{(\tau)}(0) d\tau \quad (\text{F.9})$$

or

$$\delta X_2 = \frac{1}{T_S} \left(\int_{\tau=0}^{T_S} \int_{t=0}^{T_S-\tau} S(t)(e^{bt} - 1) dt d\tau + \int_{\tau=0}^{T_S} \int_{t=T_S-\tau}^{\alpha_0} (e^{b(T_S-\tau)} - 1) S(t) dt d\tau \right) \quad (\text{F.10})$$

The overall average change of life expectancy, δX_p , for the protected group is then given by (Thomas et al., 2006c):

$$\delta X_p = \frac{X(0)\delta X_1 + T_s\delta X_2}{X(0) + T_s} \quad (\text{F.11})$$

Appendix G. The Total Judgement or J_T-value

Jones-Lee and Chilton (2017) devote the final paragraph of their Section 5 to a statement on the J_T-value (which they are incorrect to suggest applies only to nuclear hazards):

"However, it is important to appreciate the fact that since environmental effects would be dealt with quite separately from effects on health and safety in a conventional cost-benefit analysis of the type carried out by UK Government departments, such effects should not be treated as constituting part of the VPF implied by the Thomas model in any legitimate comparison with the DfT figure."

This seems to be a simple assertion of the belief held by Jones-Lee and Chilton. It appears also to embody Fallacy 2.3, the erroneous assumption that the J-value is designed to find an improved VPF figure.

Notably those authors fail to provide evidence that they have read the papers of record on the Total Judgement- or J_T-value (Thomas et al., 2010b,c, Thomas and Jones, 2010). In fact, the J_T-value provides an objective answer on how much ought to be spent on safeguards when there is a need to protect both humans and the environment from harm.

While human harm may be the predominant risk in some cases, there are other instances where an industrial protection system will need to be designed to mitigate against economic and environmental costs in addition. For example a shut-down system on a chemical plant or a nuclear reactor will protect against not only human harm but also damage to nearby plant and the spread of contamination to the environment. The BP Macondo/ Deepwater Horizon explosion and oil spill of 2010 is a prime recent example where a very bad accident on an oil and gas process plant led both to the loss of human life and to major environmental and economic harm. A properly functioning protection system would have safeguarded against both effects.

Assigning the umbrella term, "environmental costs", to cover the costs associated with environmental clean-up, evacuation of people, loss of business, damage to plant and damage to reputation, the trade-off between extra spending on the protection system and these environmental costs may be quantified using an extension to utility theory to produce a second Judgement Value, J₂₀. The result is then integrated with the J-value to produce the Total Judgement or J_T-value, which will indicate whether the total cost of the protection system is reasonable in view of its ability to protect both humans and the environment. A feature common to J, J₂₀ and J_T is that each is fully objective.

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Age, a	Life expectancy, $X(a)$, at age, a	Survival probability, $S(a)$, to age, a	Hazard rate, $h(a)$, at age a
40	42.03	0.9809	1.247×10^{-3}
80	8.92	0.6203	4.881×10^{-2}

Table 1 Parameters based on National Life Tables 2014 – 2016, combined genders

Strong intergenerational equity $(X_1, \delta X_1)$	Weak intergenerational equity $(X_p, \delta X_p)$	Mixed intergenerational equity $(X_1, \delta X_p)$
0.58	0.51	0.56

Table 2. J-value for worked example, using different assumptions on intergenerational equity

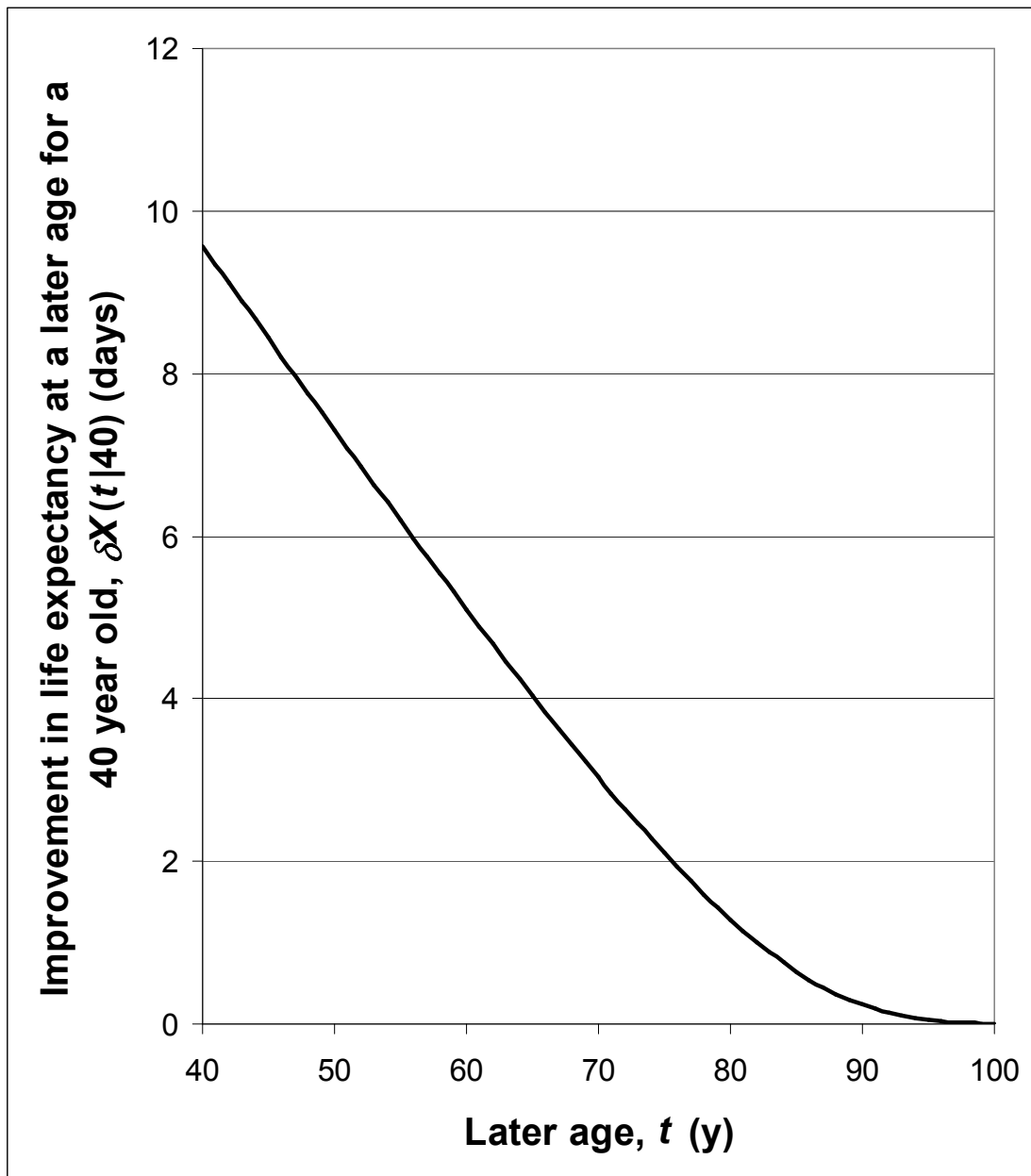


Figure 1. Improvement in life expectancy for a 40 year old when the hazard rate is reduced for one year at a later age by the same amount, $b = 6.24 \times 10^{-4}$, equivalent to 50% of the background hazard rate for a 40 year old UK citizen.

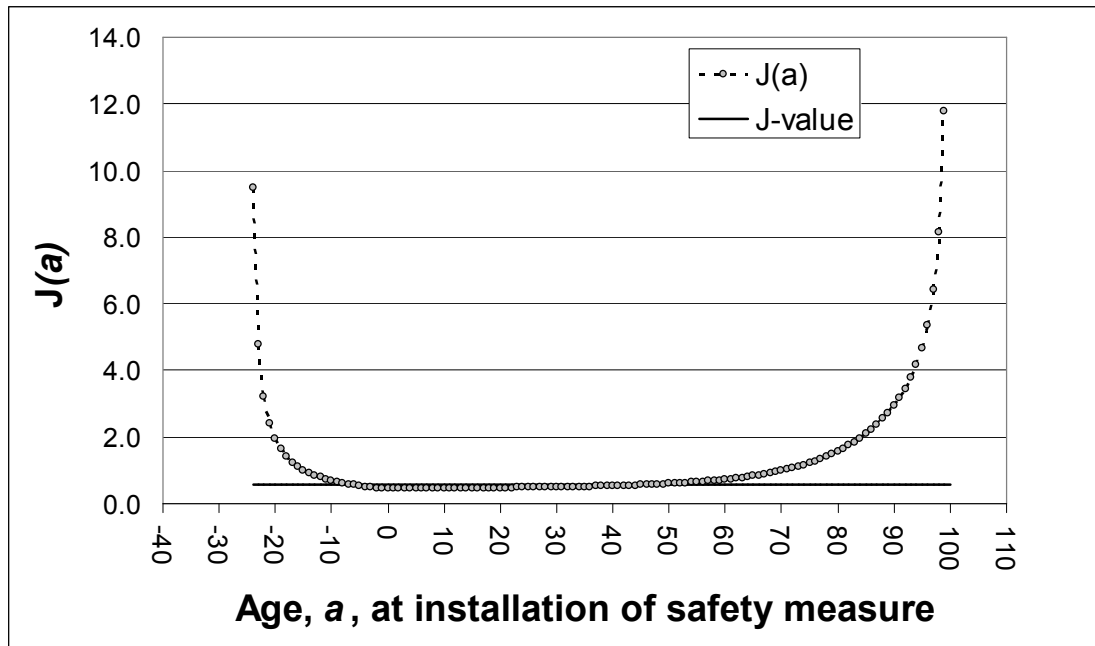


Figure 2. Individual J-value, $J(a)$, against age, a , at the time of installation of the safety measure: worked example of Appendix D. (The true J-value is marked on as a horizontal line.)